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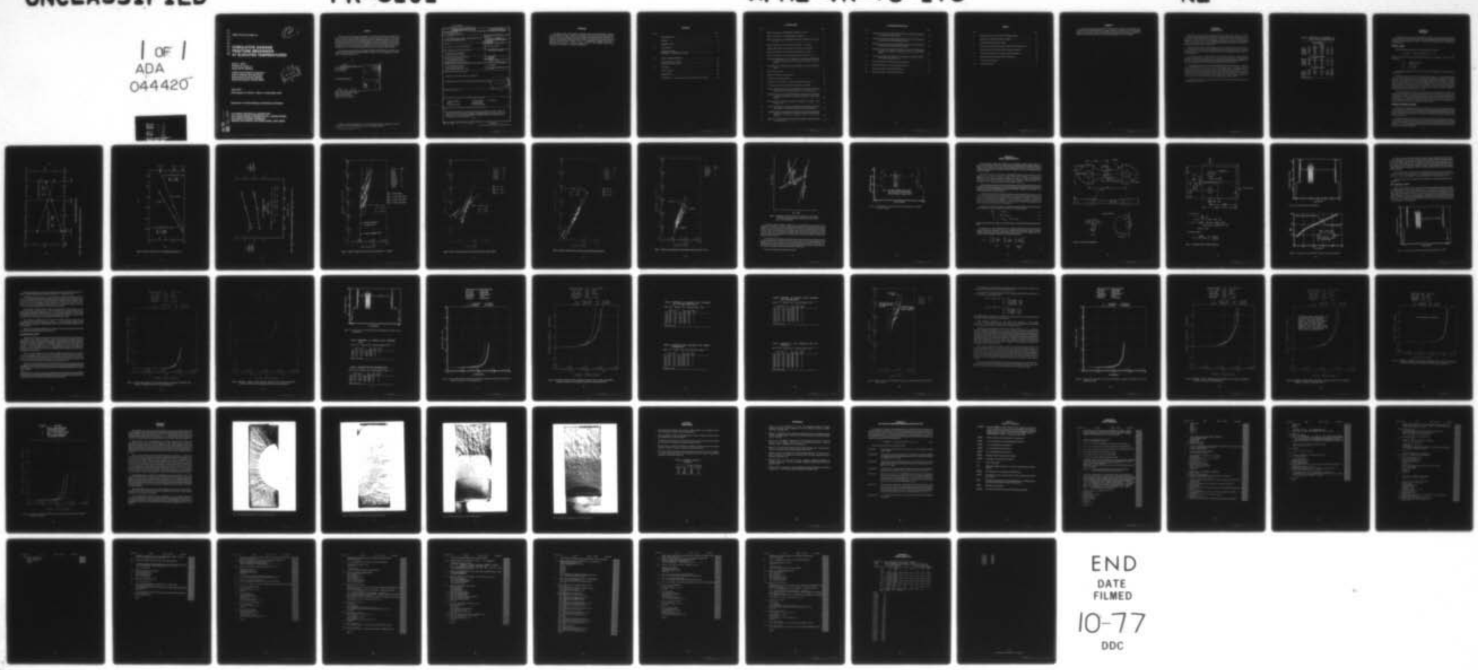

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AFML-TR-76-176 PART III

CUMULATIVE DAMAGE FRACTURE MECHANICS AT ELEVATED TEMPERATURES

David L. Sims
Charles G. Annis, Jr.
Raymond M. Wallace

United Technologies Corporation
Pratt & Whitney Aircraft Group
Government Products Division
West Palm Beach, Florida 33402

April 1977

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AIR FORCE MATERIALS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
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WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

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The interpolative hyperbolic sine model for mission-mix crack growth was used to predict the propagation histories and cycles-to-failure for two specimen geometries (compact tension and surface crack) at two temperatures (1000°F and 1200°F). Results were encouraging with an average prediction error of less than 15%.

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FOREWORD

The major portion of this work was performed under Air Force Materials Laboratory Contract F33615-75-C-5097, "Application of Advanced Fracture Mechanics at Elevated Temperatures." Dr. W. H. Reimann is the project engineer. The program is being conducted in the Materials and Mechanics Laboratories, Pratt & Whitney Aircraft Group Government Products Division, West Palm Beach, Florida. Mr. M. C. VanWanderham, Manager, Mechanics of Materials and Structures, is program manager and Mr. R. M. Wallace, Group Leader, Component Life Analysis, is the principal investigator.

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SUMMARY

This report provides a demonstration of an interpolative model for crack propagation life predictions at elevated temperature. The model is based on the hyperbolic sine. Two specimen geometries were tested under stress-temperature-time spectra that represent advanced gas turbine disk operating conditions. A critique of model predictive ability is presented.

SECTION I
INTRODUCTION

Historically, the methods for predicting low-cycle fatigue (LCF) life have produced conservative underestimations of total useful life, resulting in costly early replacement of LCF limited gas turbine engine rotating components. Accurate total LCF life predictions must consider (1) the initiation of an actively propagating macrocrack, (2) fatigue crack propagation under constant maximum load, and (3) deviations in propagation behavior (acceleration and/or retardation) caused by major load excursions.

Engine hardware operates under complicated stress-time-temperature spectra, but laboratory testing must be done at selected conditions because of cost and time limitations. To describe crack propagation at conditions where test data does not exist, an interpolative crack propagation model is necessary.

This report presents a demonstration of an interpolative model for crack propagation behavior at elevated temperatures, which is based on the hyperbolic sine. Propagation life predictions using this model are made for two crack geometries tested under stress-temperature-time spectra representative of advanced gas turbine disk operating conditions.

All fracture mechanics evaluations were performed on an advanced nickel-base turbine disk alloy, GATORIZED™ IN-100, used in the F100 turbofan engine. Specimens specifically designated to this contract were machined from heat BANQ-499, but a significant amount of crack propagation data existed for this alloy prior to the start of this program and is also used in the analyses. Heat treatment consists of solutionization at 2050°F, stabilization at 1600°F and 1800°F, and precipitation hardening at 1200°F and 1400°F. Typical chemical composition is 0.07C-12.4Cr-18.5Co-3.2Mo-4.32Ti-4.98Al-0.78V-0.02B-0.06Zr - balance nickel.

Tensile, stress rupture, and creep test results for two forgings from this heat (499-A2A and 499-A2B) are given in Table 1.

TABLE 1. MECHANICAL PROPERTIES OF TWO IN-100 PANCAKE FORGINGS

Tensile Properties

Disk No.	*Temp (°F)*	*Yield Strength (ksi)*	*Ultimate Tensile Strength (ksi)*	*EL %*	*RA %*
499-A2A	RT	164.5	232.4	22.0	22.2
499-A2A	1300	157.2	177.0	14.0	22.3
499-A2B	RT	164.9	232.0	22.0	21.5
499-A2B	1300	156.0	179.1	14.7	16.4

Stress Rupture Properties

Disk No.	*Temp* (°F)	*Stress* (ksi)	*Life* (hr)	*EL* %	*RA* %
499-A2A	1350	95	28.0	10.6	15.9
499-A2B	1350	95	18.0	8.1	15.6
499-A2B	1350	95	19.5	8.0	8.2
499-A2B	1350	95	19.3	6.7	12.9

Creep Rupture Properties

Disk No.	*Temp (°F)*	*Stress (ksi)*	*0.1% (hr)*	*0.2% (hr)*	*Total Life (hr)*
499-A2A	1300	80	—	175.5	233.2*
499-A2B	1300	80	114.5	142.5	143.2*

*Test Discontinued.

SECTION II
THE MODEL

An interpolative model has been developed for the analysis of elevated temperature fatigue crack propagation data. This model is used to describe the parametric effects of four fundamental influences on crack propagation: frequency (ν), stress ratio (R), temperature (T), and major load excursion effects.

GENERAL MODEL

This interpolative model is based on the hyperbolic sine equation,

$$\log(da/dN) = C_1 \sinh\left(C_2 (\log(\Delta K) + C_3)\right) + C_4 \qquad (1)$$

where the coefficients are simple empirical functions of test frequency, stress ratio, and temperature:

$$\begin{aligned} C_1 &= \text{material constant} \\ C_2 &= f_2 (R, \nu, T) \\ C_3 &= f_3 (C_4, \nu, R) \\ C_4 &= f_4 (\nu, R, T) \end{aligned}$$

A more complete description of the model is presented in Reference 1. The salient features are given here.

It has been shown that for IN-100, the coefficients in equation 1 can be simple empirical functions of cyclic frequency, stress ratio, and temperature. Experience indicates that, for a given material, C_1 can be fixed without adversely affecting model flexibility. For IN-100, C_1 has a fixed value of 0.5. The coefficients C_2 and C_4 are logarithmic functions of cycle duration, $1/\nu$, (Figure 1) and C_3 exhibits linear variation with log (1-R) (Figure 2). The coefficients C_2 and C_4 are also linear functions of temperatures (Figure 3), from 1000°F to 1350°F. C_3 does not change with temperature because the correlation line is vertical. The relationships for frequency, stress ratio, and temperature are given in Figures 4, 5, 6, and 7.

The simple relationships previously discussed describe crack propagation of IN-100, at any stress ratio and frequency for temperatures between 1000°F to 1350°F in air conditions. The computational procedure is schematically represented in Figure 8. First locate the coefficients on the 1200°F, R = 0.1 frequency model, position 1. Second, account for stress ratio effects by moving along a stress ratio model to position 2. Finally, C_2 and C_4 for the desired temperature are determined using $\partial C/\partial T$ from the temperature model, position 3.

EMPIRICAL SYNERGISTIC MODEL

The majority of crack growth studies are performed using constant amplitude loading. These tests are not representative of gas turbine rotating component operating conditions, which include complex stress-time-temperature histories. It is expected that crack growth relationships will be complicated by mission mix cyclic conditions that result from throttle excursions, disk-blade interactions, etc.

A generalized predictive model must account for the effects of load sequence. A mission is first analyzed to determine if principles of linear superposition of damage were applicable. If they were not, then synergism must be considered. Since interaction effects are sensitive to the loading sequence, accurate crack propagation models can only be expected when realistic load-time histories are simulated (Reference 2).

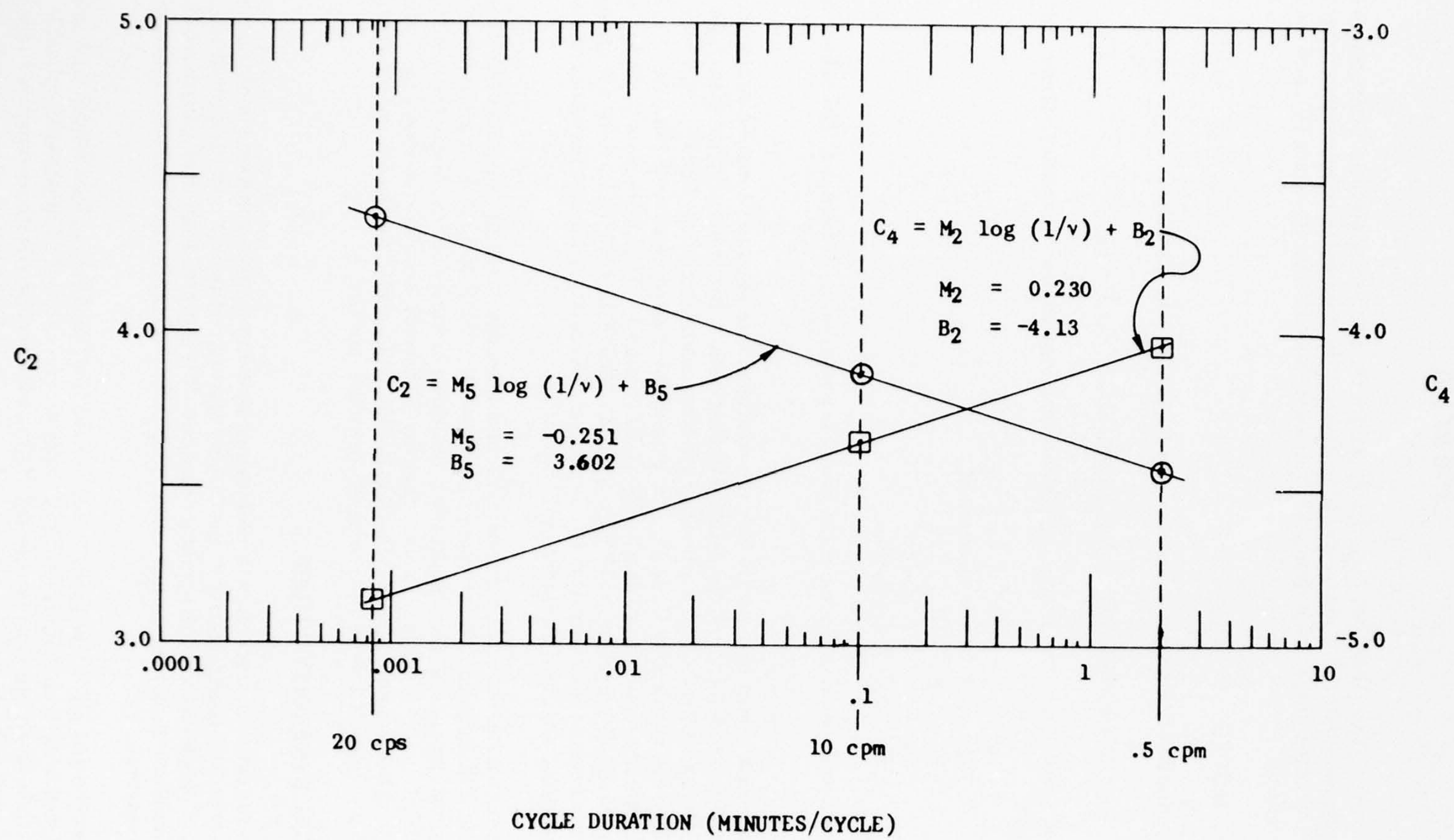


Figure 1. Effect of Frequency on SINH Model Coefficients C_2 and C_4

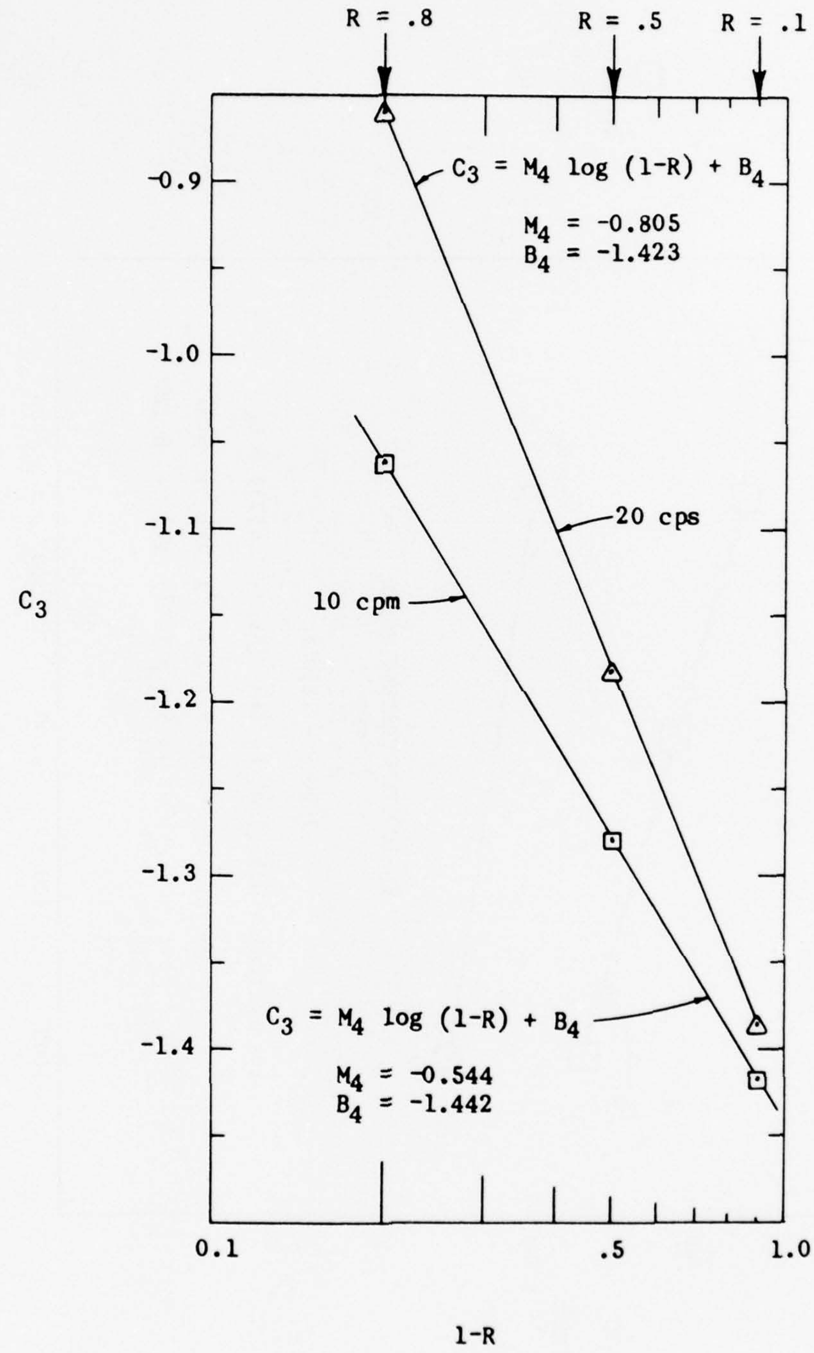


Figure 2. Effect of Stress Ratio, R, on SINH Model Coefficient, C_3

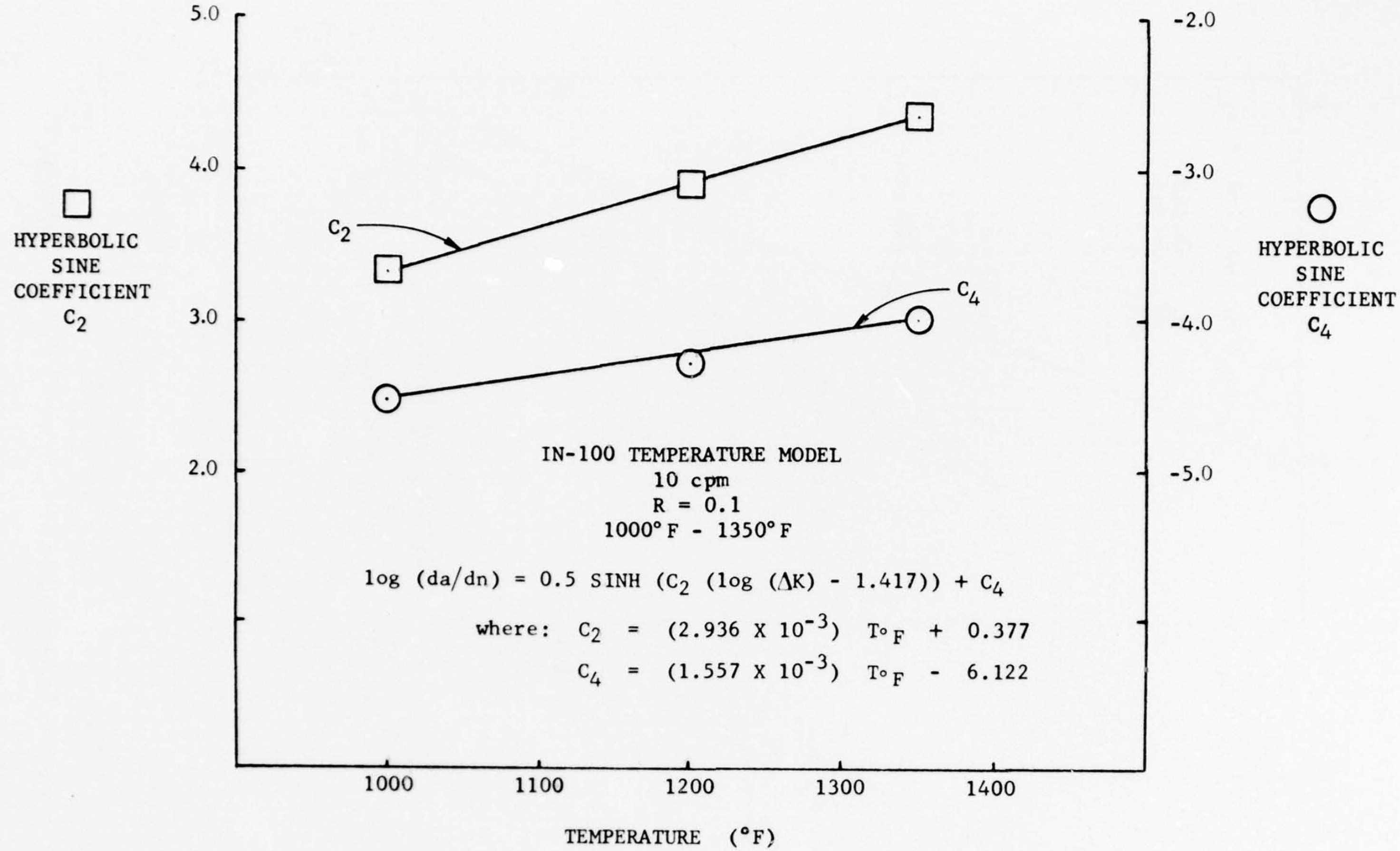


Figure 3. Effect of Temperature on SINH Model Coefficients (1000°F-1350°F, 10 cpm, R = 0.1)

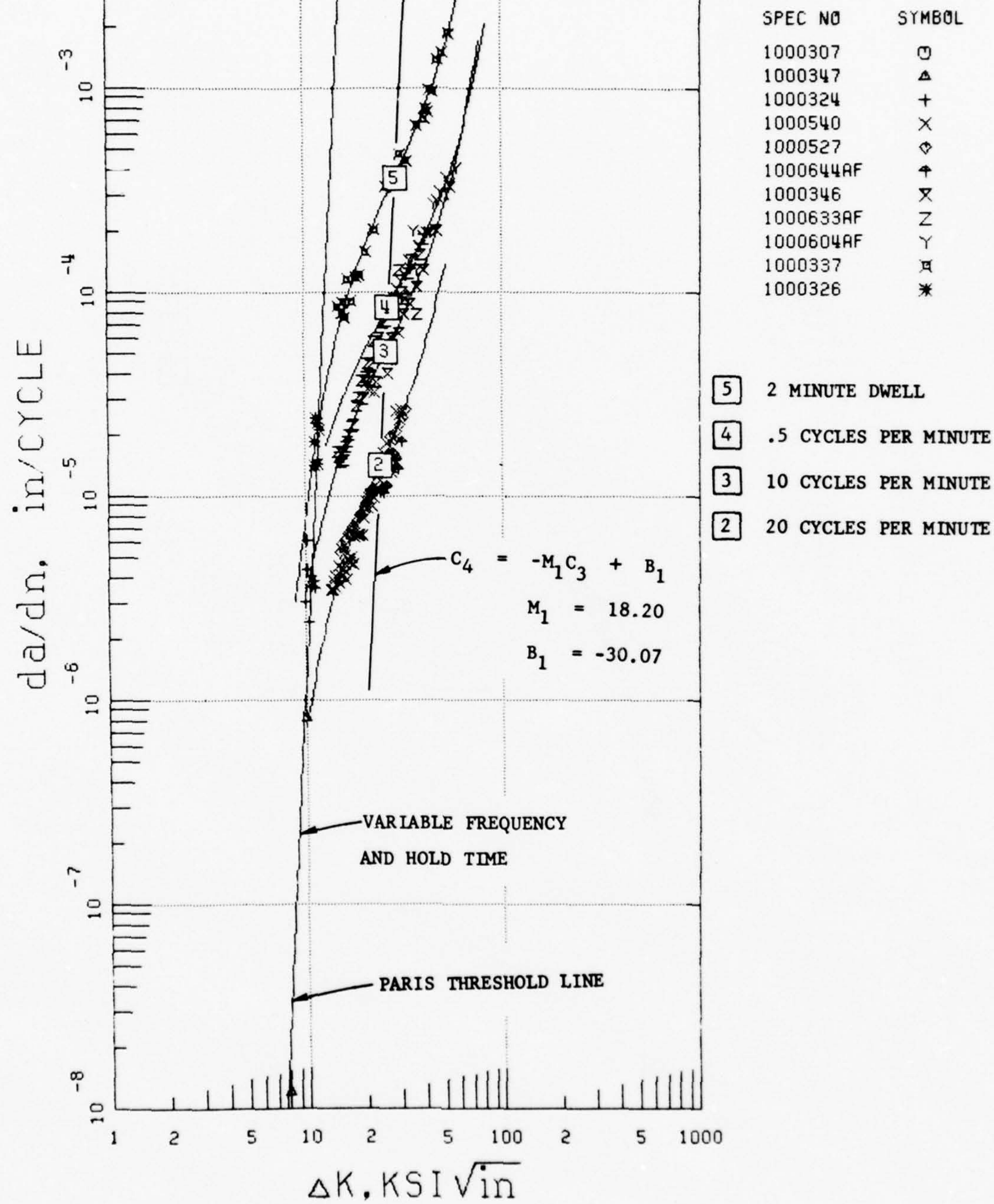


Figure 4. Effect of Frequency on Crack Growth in IN-100 R = 0.1, 1200°F

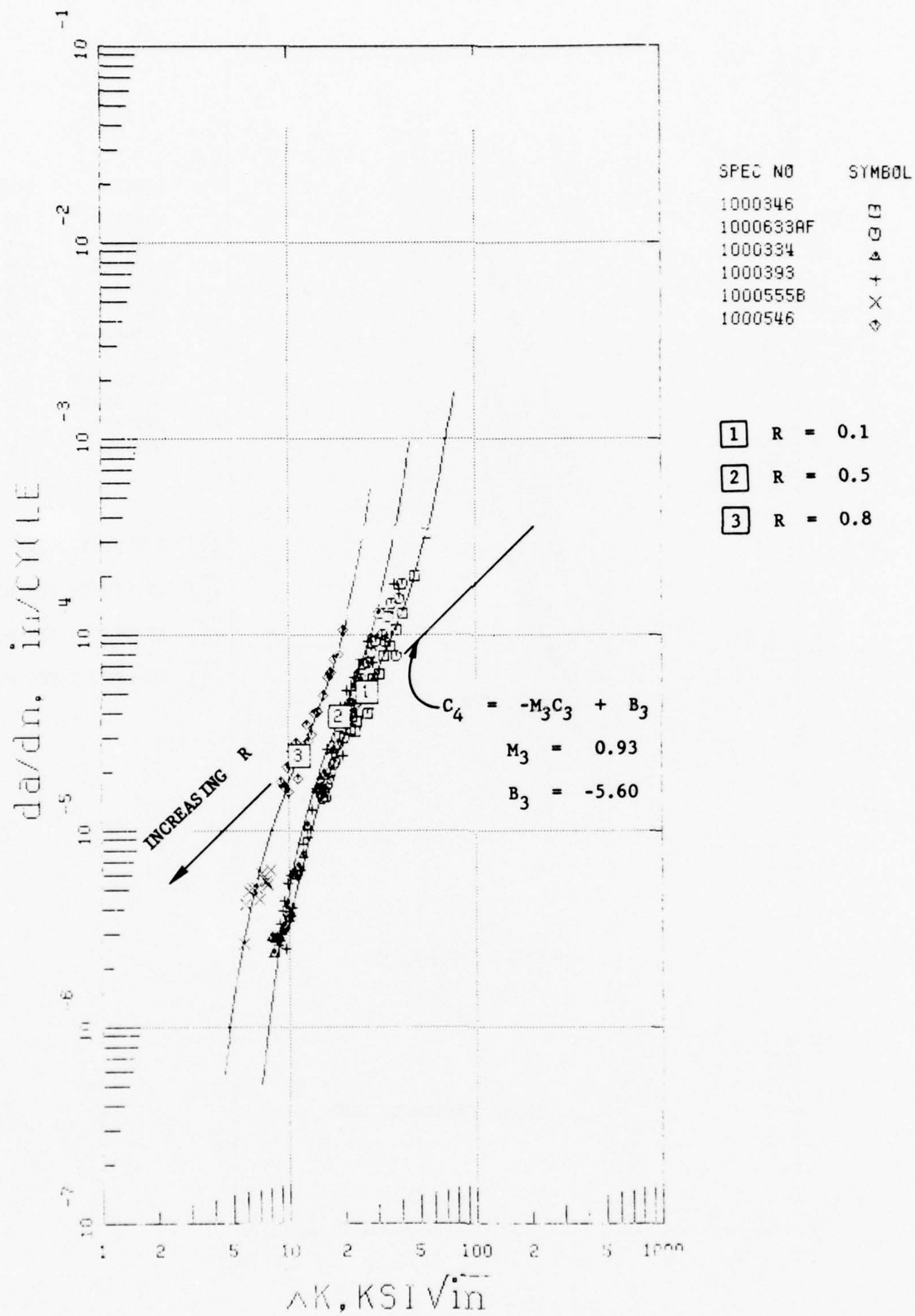


Figure 5. Effect of Stress Ratio on Crack Growth in IN-100, 10 cpm, 1200°F

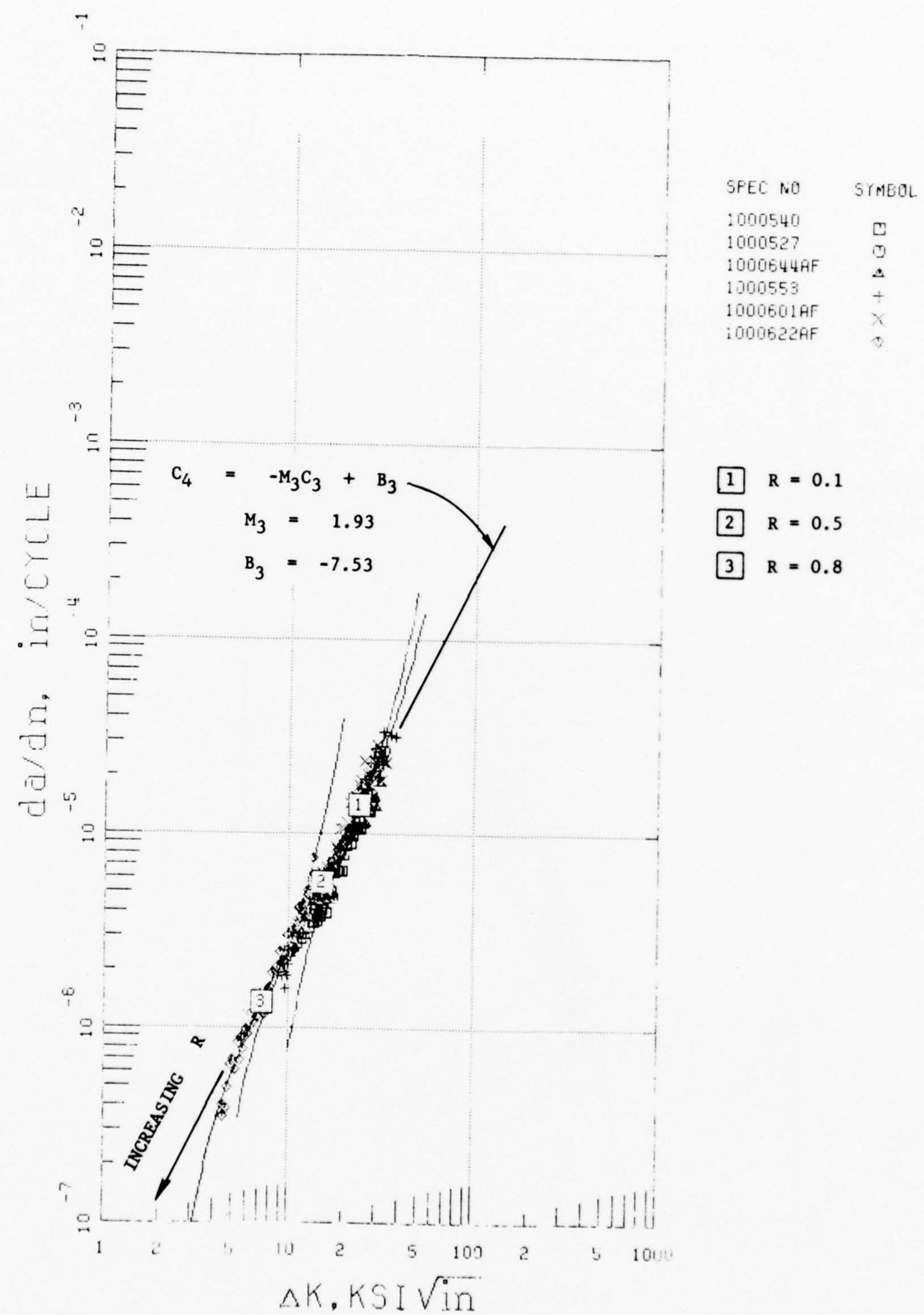


Figure 6. Effect of Stress Ratio on Crack Growth in IN-100 20 cps, 1200°F

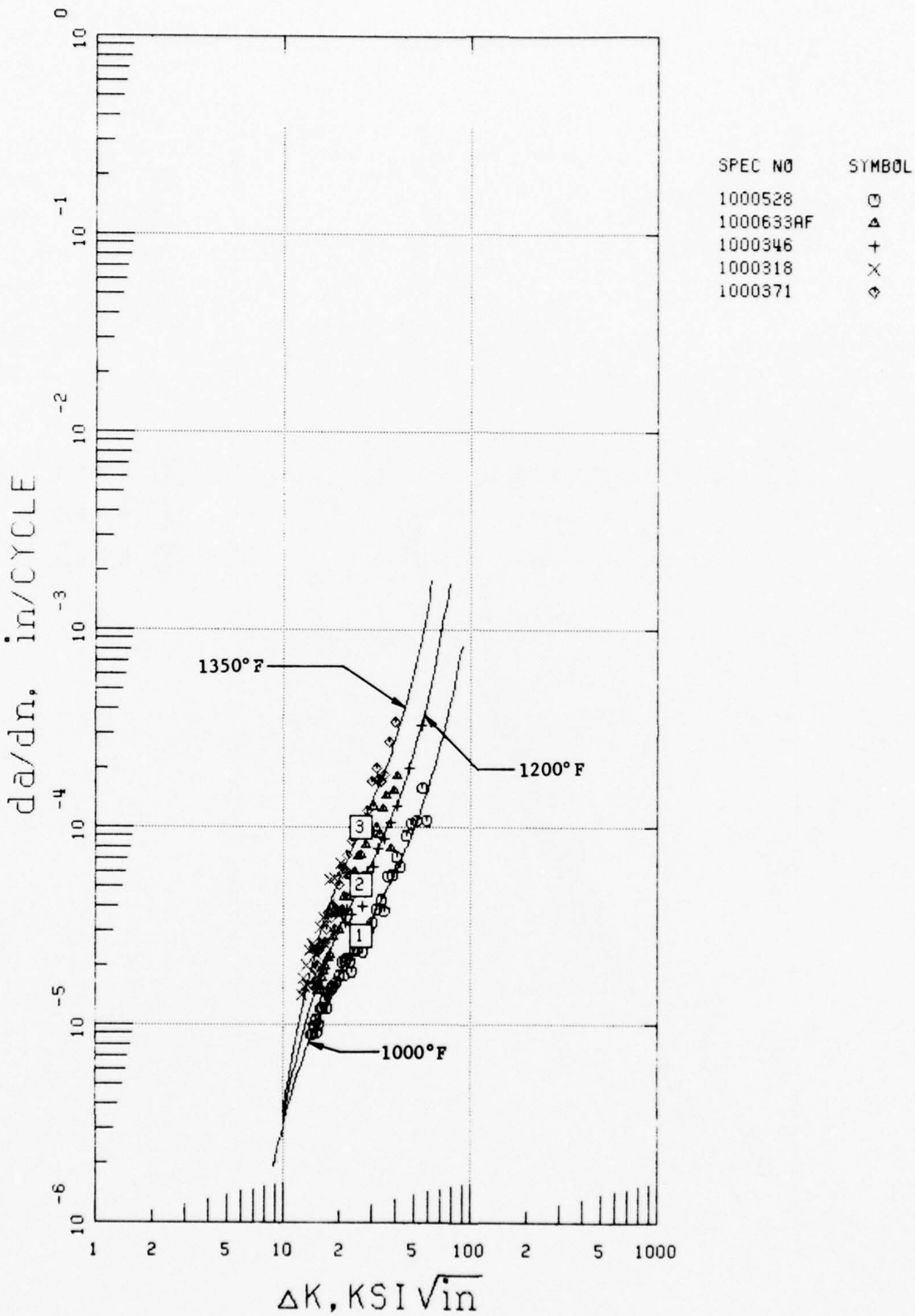


Figure 7. Effect of Temperature on Crack Growth in IN-100, 10 cpm, R = 0.1

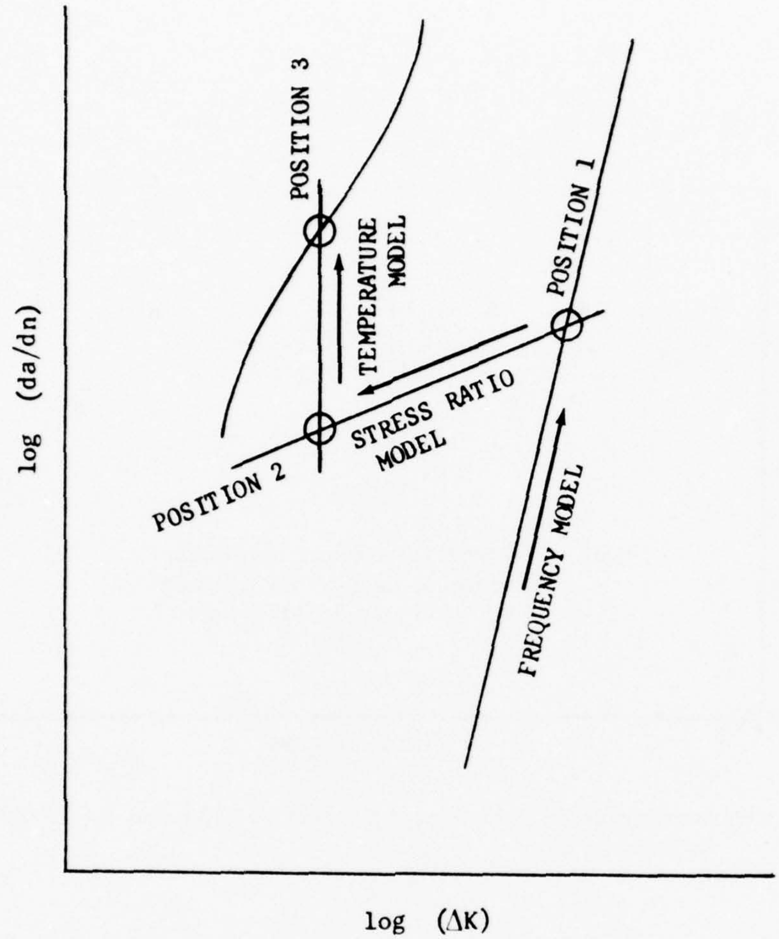


Figure 8. Schematic Representation of the Method for Determining SINH Model Coefficients Representing Any Frequency, Stress Ratio, and Temperature

If a realistic load-time mission is broken down into small segments, the segments can be represented by synergistic empirical models. Linear superposition of damage for each mission segment can be used to predict crack propagation life. Care must be taken when dividing the mission to ensure minimum segment interaction at the division points. For example, major load excursions affect the cycles following to a greater degree than they are affected by those which proceed, mission division should be immediately prior to an excursion rather than after. (See Figure 9, segments 4 and 7.)

The interpolative hyperbolic sine model is ideal for representing crack propagation data of complex missions since it requires only a minimum number of tests. Both acceleration and retardation of the crack growth rate caused by overloads can be accurately represented. The SINH does not attempt to model the micromechanical physical deformation near the crack tip, but empirically describes the macroscopic behavior of the crack.

This model is discussed in detail in Section III.

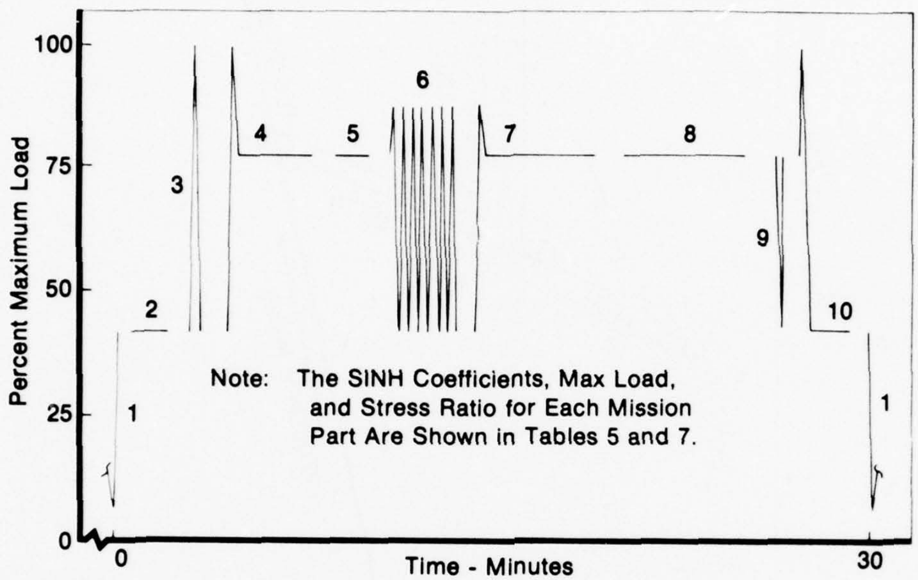


Figure 9. Incrementation of Simulated Turbine Disk Mission for 1200°F, Retardation Model

SECTION III
MODEL DEMONSTRATION

Two geometries, surface flaw (Figure 10) and modified compact tension (Figure 11) specimens, were tested at the same two temperatures to demonstrate the interpolative predictive ability of the hyperbolic sine model under simulated advanced gas turbine disk operating conditions. Load vs time for the mission is shown in Figure 12. The two test temperatures were 1000°F and 1200°F.

The stress intensity (K) solution for the Modified Compact Tension (MCT) specimen (Reference 3) is shown in Figure 11. The surface flaw specimen was designed by P&WA/Commercial Products Division (Reference 4). Figure 13 compares the analytically determined surface flaw K-solution and the solution calculated using handbook values (Reference 5) for a semicircular surface flaw under uniform tension.

All testing was performed on servocontrolled hydraulic rigs. MCT specimens were tested on an MTS rig with digital computer control and the surface crack specimens were tested on a rig designed and built by P&WA/Florida, using DATATRAK® controllers. All specimens were heated using clamshell resistance furnaces.

A mathematical model describing crack propagation has utilitarian value only insofar as it can be used in life prediction; the overall accuracy of a model can be measured by the accuracy of the resulting prediction. To provide a basis for comparing the accuracy of various life predictions, a simple correlative parameter is used, N_{pred}/N_{act}, the quotient of predicted and actual cyclic lives. Ideally, this quotient is 1.0 and decimal deviations from the ideal can be quickly interpreted as percent error of the prediction.

A simple cycle-by-cycle (or mission-by-mission) integration is used to sum the incremental crack advances, da, which comprise the cyclic life, N, (or mission life, M).

$$da/dN = f\,(a,\ \Delta K\ .....) \tag{2}$$

$$dN = da/f\,(a,\ \Delta K\ .....) \tag{3}$$

$$N = \int dN = \int_{a_i}^{a_f} da/f\,(a,\ \Delta K\ .....) \tag{4}$$

Because this integral can be difficult to evaluate directly, computerized numerical methods are used.

In equation 2, f (ΔK) is represented by any empirical model that accurately predicts the average instantaneous crack growth rate for a calculated stress intensity. Linear cumulative damage within a mission spectrum is determined using cycle-by-cycle (or unit time-by-unit time) integration and subsequent summation of individual components.

$$N_{missions} = \left[\underbrace{\sum_{0}^{N_c} \frac{da}{f(\Delta K_1)}}_{\text{cyclic}} + \underbrace{\sum_{0}^{N_d} \frac{da}{f(\Delta K_2)}}_{\text{dwell}} + \underbrace{\sum_{0}^{t} \frac{da}{f(K_3)}}_{\text{sustained load}} \right]_{a_i}^{a_f} \tag{5}$$

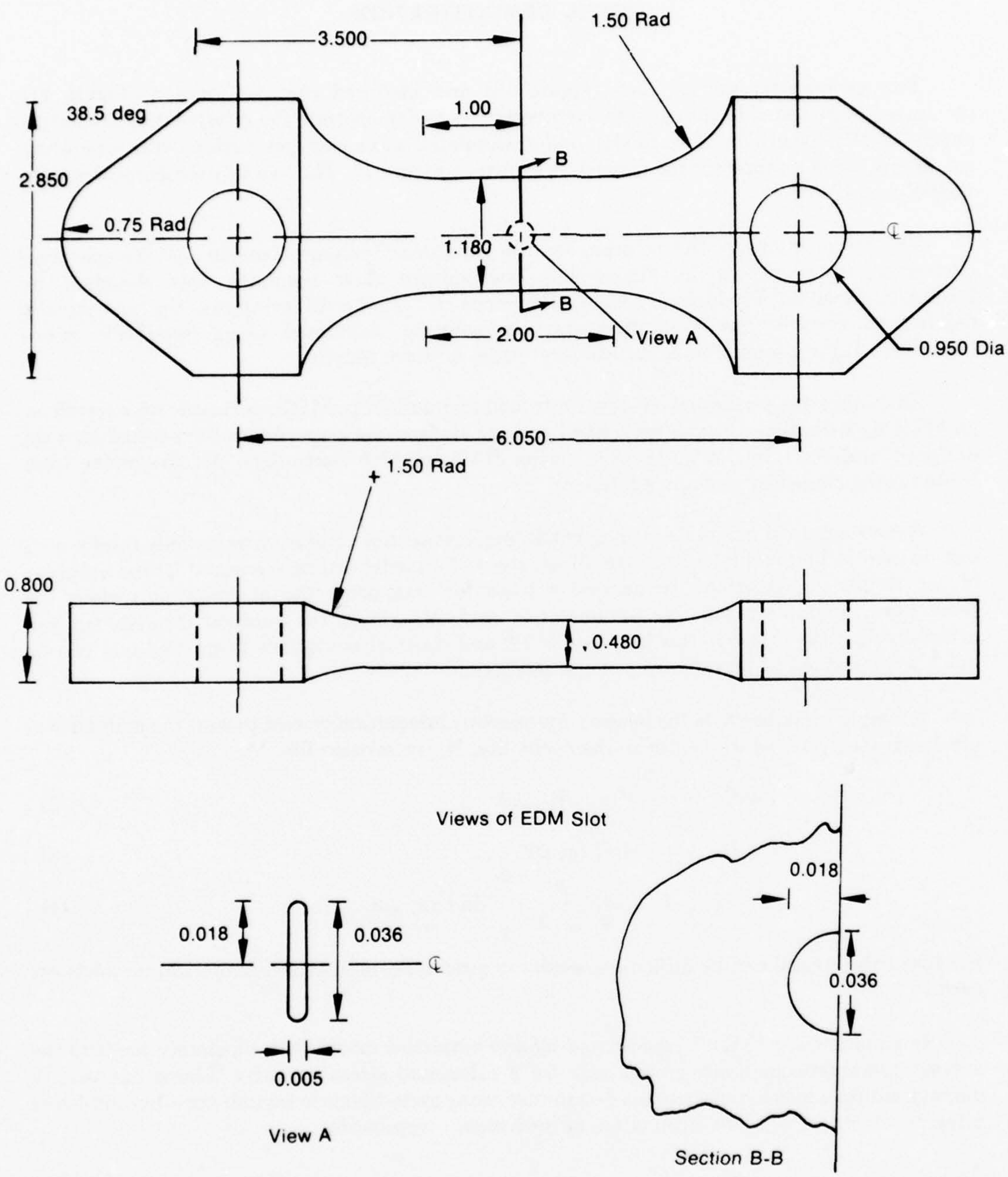
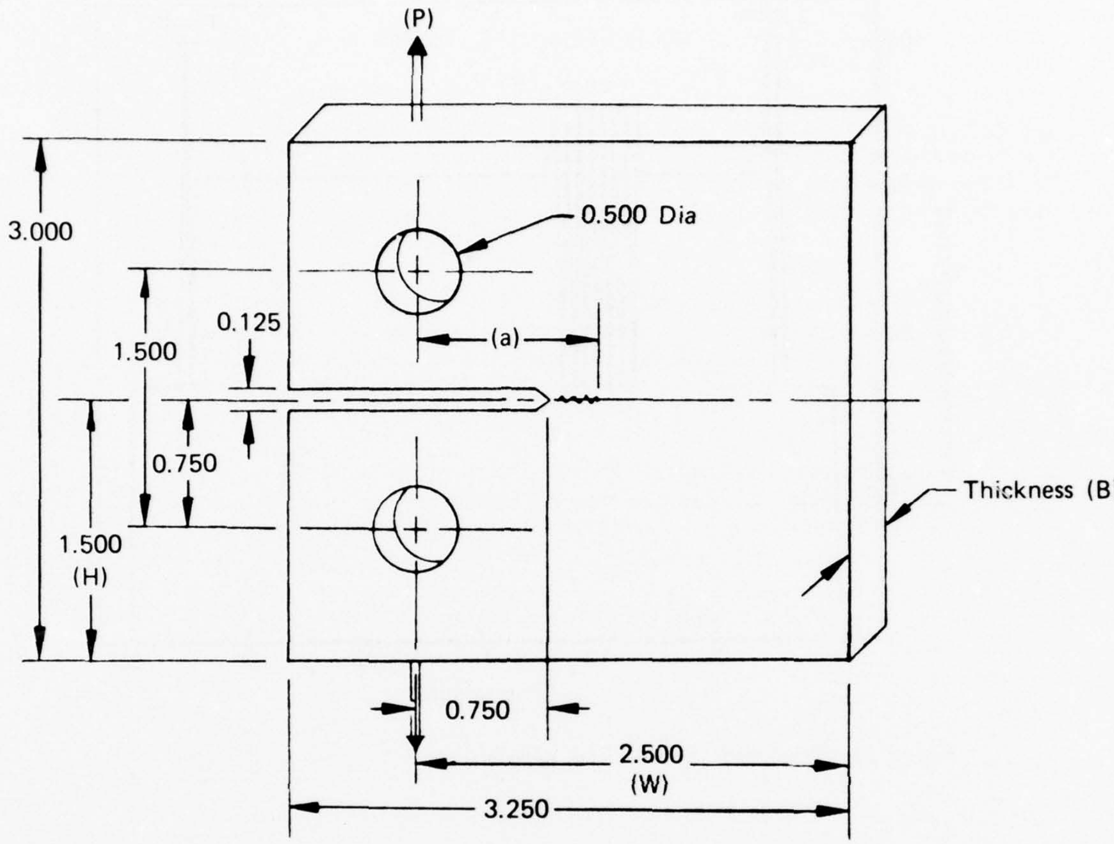


Figure 10. Surface Flaw Specimen



K Calibration:

$$K = Y \frac{P}{BW} \sqrt{a}$$

for a/w = 0.3 - 0.7; H/W - H/W = 0.6

$$Y = f\,(a/w) = [0.2960 - 1.855\,(a/w) + 6.557\,(a/w)^2 - 10.17\,(a/w)^3 + 6.389\,(a/w)^4]\ 10^2$$

Accuracy: 0.5%

Net Section Stress;

$$\sigma_{Net} = \frac{KW^{1/2}}{f\,(a/w)\,(w\text{-}a)} \left[1 + \frac{3(w+a)}{(w\text{-}a)}\right]$$

Figure 11. Modified Compact Tension Specimen

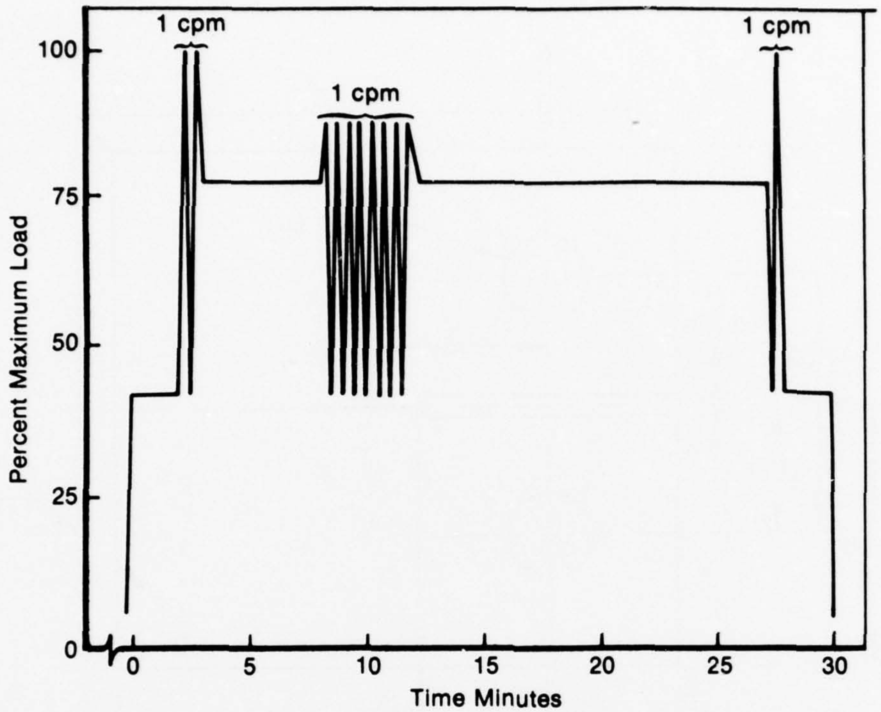


Figure 12. Simulated Turbine Disk Mission

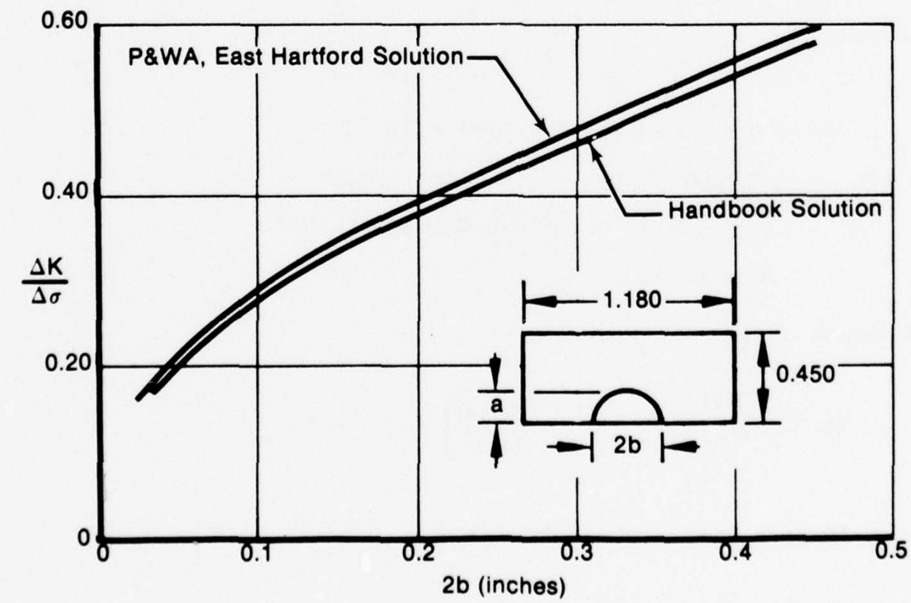


Figure 13. Comparison of K-Calibration Curves for Surface Flaw Specimen

A computer program has been developed to perform crack propagation life analyses using (1) linear superposition of individual components within a mission, and (2) empirical synergistic models. The program predicts the life of specimens tested under variable amplitude cyclic, cyclic-dwell, and/or dwell conditions. Currently, a total of twenty separate segments within a given mission can be incrementally integrated to predict crack propagation life. The empirical inputs describing crack propagation within the mission segments are the interpolative SINH model coefficients.

The program provides, as output, computer drawn (GOULD) plots of the actual a vs N data (included as input) and the predicted a vs N relationship. This allows direct comparison of predicted vs actual data. Appendix C shows an example of the complete program input.

Instructions for use of the program are given in Appendix A. The actual program is listed in Appendix B.

LIFE ANALYSIS AT 1000°F

Figure 14 shows the test mission segmented into appropriate regions for analysis at 1000°F. Two assumptions are made in this analysis. First, it is assumed that sustained load crack propagation does not occur under the test conditions. This is based on the observation that there was no significant difference between 10 cpm and 2-minute dwell crack propagation rates at 1000°F (Reference 1). Second, it is assumed that no synergism occurs. This is necessary because there are no data yet available on overload effects at 1000°F for IN-100. Therefore, the predictions are based on interpolations of the SINH model for the cyclic portions, assuming sustained load crack propagation to be insignificant.

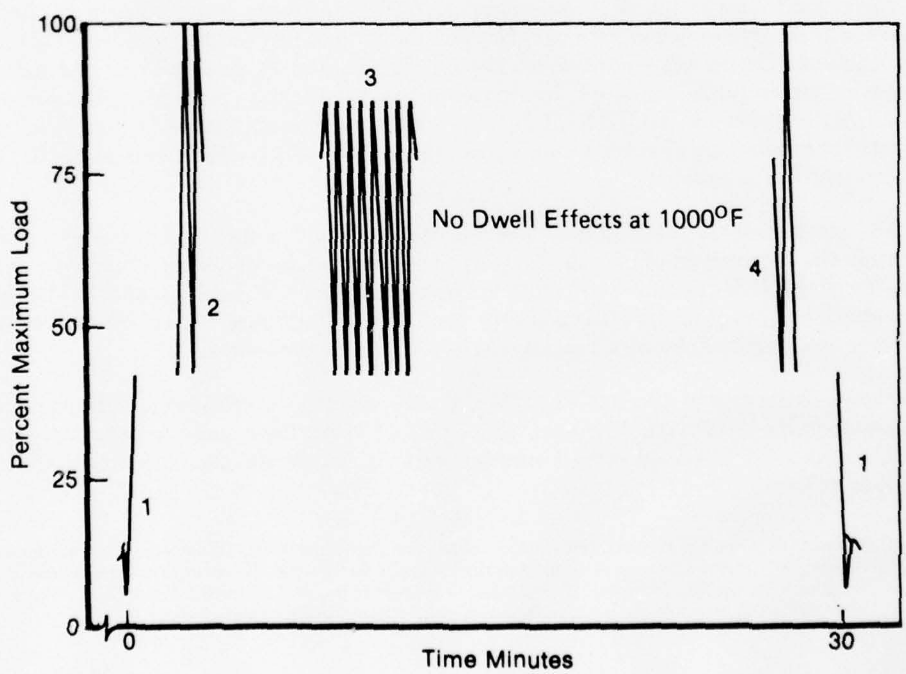


Figure 14. Incrementation of Simulated Turbine Disk Mission for 1000°F

All predicted mission lives, and a vs N relationships were computed and published prior to testing, in PWA FR-7906 Quarterly Progress Narrative No. 6, 15 September 1976.

The SINH equation for 10 cpm, R = 0.1 crack growth rate at 1000°F was used as the starting point for calculating the hyperbolic sine coefficients for each segment of the 1000°F mission. It is assumed that stress ratio effects at 1000°F are similar to stress ratio effects at 1200°F. Since frequency effects on crack propagation rates would be less at 1000°F than at 1200°F because of reduced environmental degradation (oxidation, Reference 1), the difference between 1 cpm and 10 cpm crack growth rates at 1000°F is considered to be negligible.

The prediction, published before testing, for the surface flaw specimen (Figure 15) was conservative. The accuracy (N_{pred}/N_{actual} = 0.91) of the prediction is encouraging, since only two tests had been conducted at 1000°F (10 cpm, R = 0.1 and 2-minute dwell, R = 0.1) prior to this mix mission test. The actual a vs N data has the same shape as the predicted a vs N relationship indicating both an accurate K-solution and an accurate predictive model.

The prediction, published prior to testing, for the MCT specimen (Figure 16) was anticonservative. The accuracy was good (N_{pred}/N_{actual} = 1.12), and the shape of the actual data is close to the predicted shape. No explanation is offered for the anticonservative behavior of the MCT specimen.

Tables 2 and 3 present the SINH equation coefficients, and other parameters for the surface flaw specimen and the MCT specimen, respectively.

LIFE ANALYSIS AT 1200°F

Figure 17 shows the test mission segmented into appropriate parts for linear superposition (no synergistic interactions) life analysis at 1200°F. The interpolative hyperbolic sine model (Reference 1) provides crack growth equations for exact operating conditions during each segment of the mission. Linear superposition of the damage caused by each part of the mission (no synergistic interactions) resulted in conservative predictions for both the surface flaw (N_{pred}/N_{actual} = 0.57) and MCT (N_{pred}/N_{actual} = 0.42) specimens. Figures 18 and 19 respectively illustrate the results, and Tables 4 and 5 give the SINH equation coefficients and other data for each segment of the mission.

The more realistic prediction uses a retardation SINH model (Reference 6) to represent accurately the damage caused by the mission. The retardation model is segmented as shown in Figure 9 with SINH equation coefficients for each part shown in Tables 6 and 7. The retardant or accelerative effect of overloads (mission parts 4, 7 and 10, Figure 9) has been modeled for the first 5 minutes of sustained load crack growth using the hyperbolic sine.*

Figure 20 illustrates the effects of 25% and 50% repetitive overloads on sustained load crack propagation in IN-100 at 1200°F. Sustained load data (no overloads) are compared with sustained load data plus a 25% overload every 2 minutes, and with sustained load data plus a 50% overload every 2 minutes.

---

*While the mission data used 2-minute dwell periods, it has been shown previously (Reference 7) there is no significant difference between 1-, 2-, and 5-minute constant amplitude tensile dwell cycles. However, there is a difference between 5-minute and 10-minute dwells. Therefore, it is postulated that there is a critical incubation period (between 5 and 10 minutes) before oxidation becomes the crack propagation rate controlling mechanism.

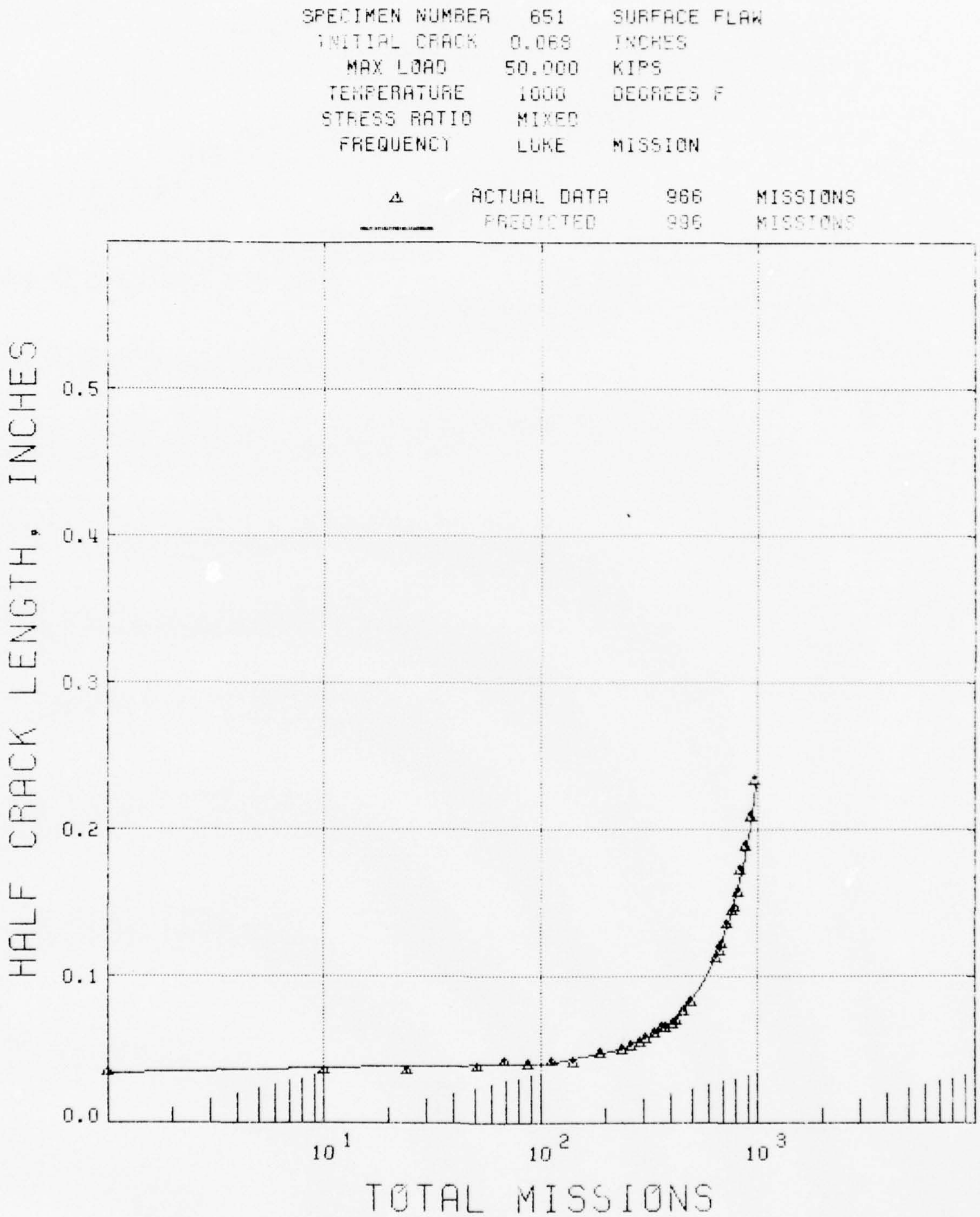


Figure 15. Surface Flaw Specimen Crack Growth Prediction, Interpolative Hyperbolic Sine Model, No Synergistic Interaction, Linear Superposition

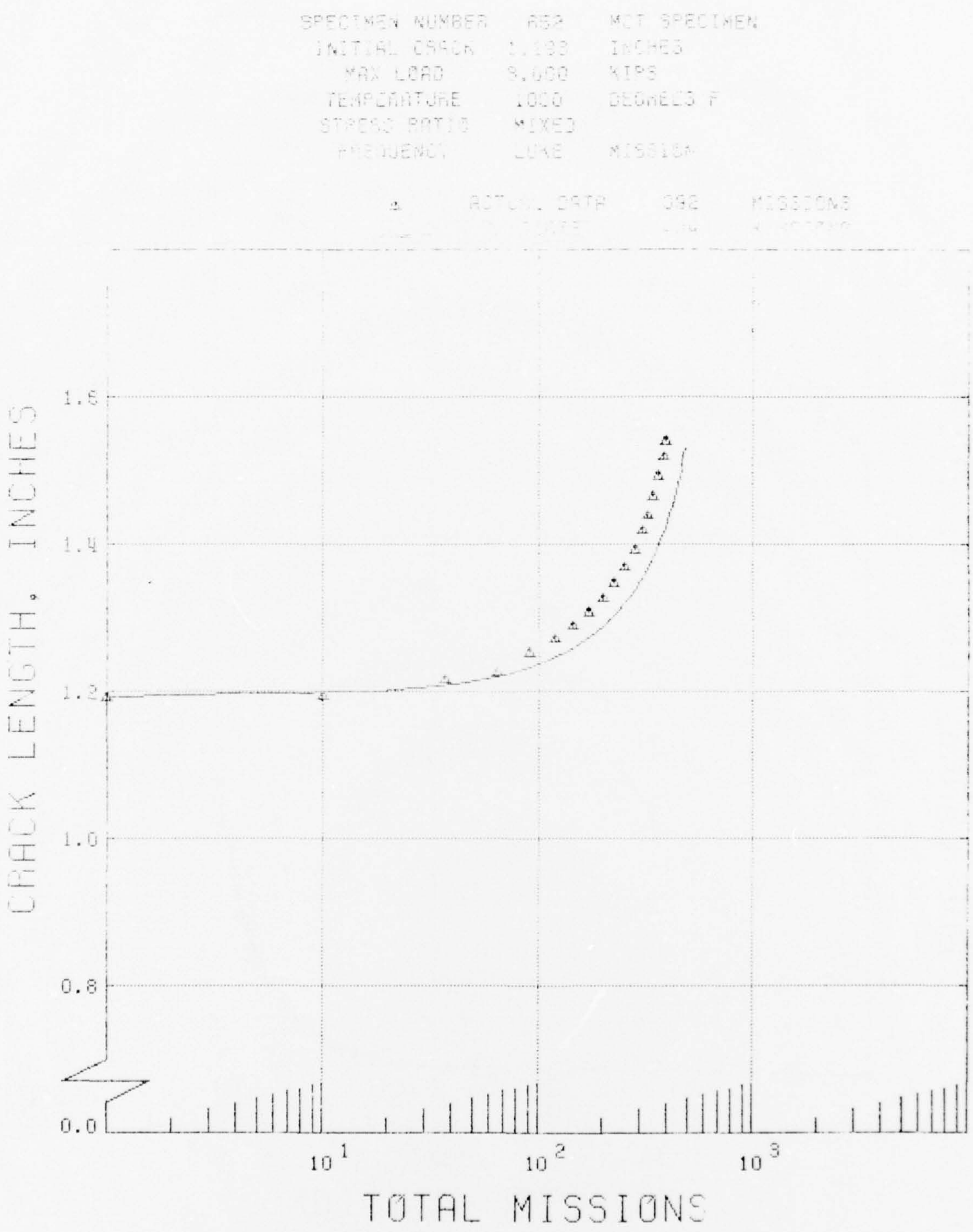


Figure 16. Modified Compact Tension Specimen Predicted Crack Growth, Interpolative Hyperbolic Sine Model, No Synergistic Interaction, Linear Superposition

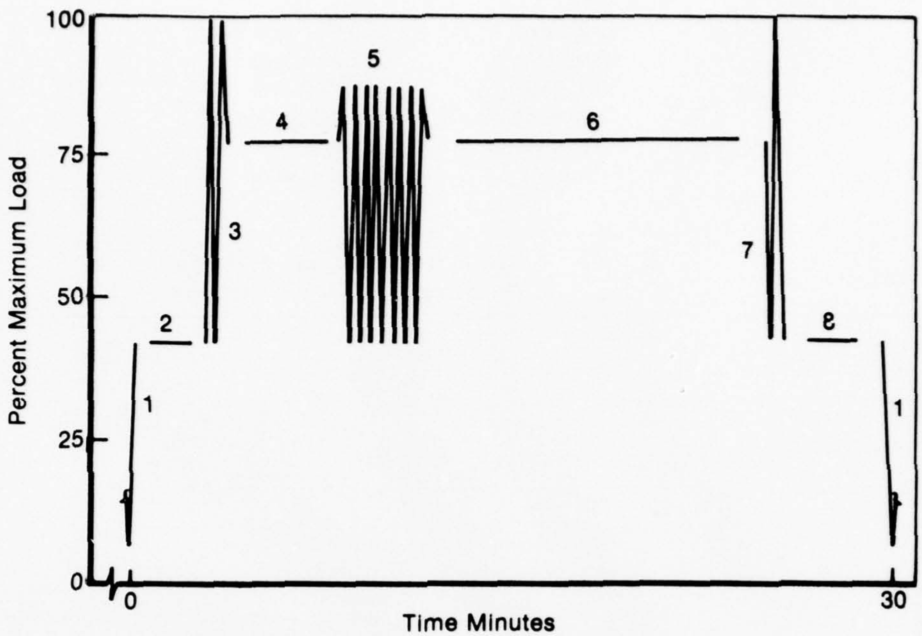


Figure 17. Incrementation of Simulated Turbine Disk Mission for 1200°F, Linear Superposition

TABLE 2. SPECIMEN 651 SURFACE FLAW SPECIMEN 1000°F

Width = 1.174 Thickness = 0.428 Initial Crack Length = 0.030

SINH Coefficients				*Max**	*Stress*	*Mission*
C1	*C2*	*C3*	*C4*	*Load*	*Ratio*	*Part*
0.500	3.320	−1.417	−4.550	23.000	0.1	1
0.500	4.151	−1.278	−4.679	50.000	0.5	2
0.500	4.151	−1.278	−4.679	44.500	0.5	3
0.500	4.151	−1.278	−4.679	50.000	0.5	4

*All loads are in Kips.

TABLE 3. SPECIMEN 652 MCT SPECIMEN 1000°F

Width = 2.503 Thickness = 0.847 Initial Crack Length = 1.000

SINH Coefficients				*Max**	*Stress*	*Mission*
C1	*C2*	*C3*	*C4*	*Load*	*Ratio*	*Part*
0.500	3.320	−1.417	−4.550	4.600	0.1	1
0.500	4.151	−1.278	−4.679	10.000	0.5	2
0.500	4.151	−1.278	−4.679	8.900	0.5	3
0.500	4.151	−1.278	−4.679	10.000	0.5	4

*All loads are in Kips

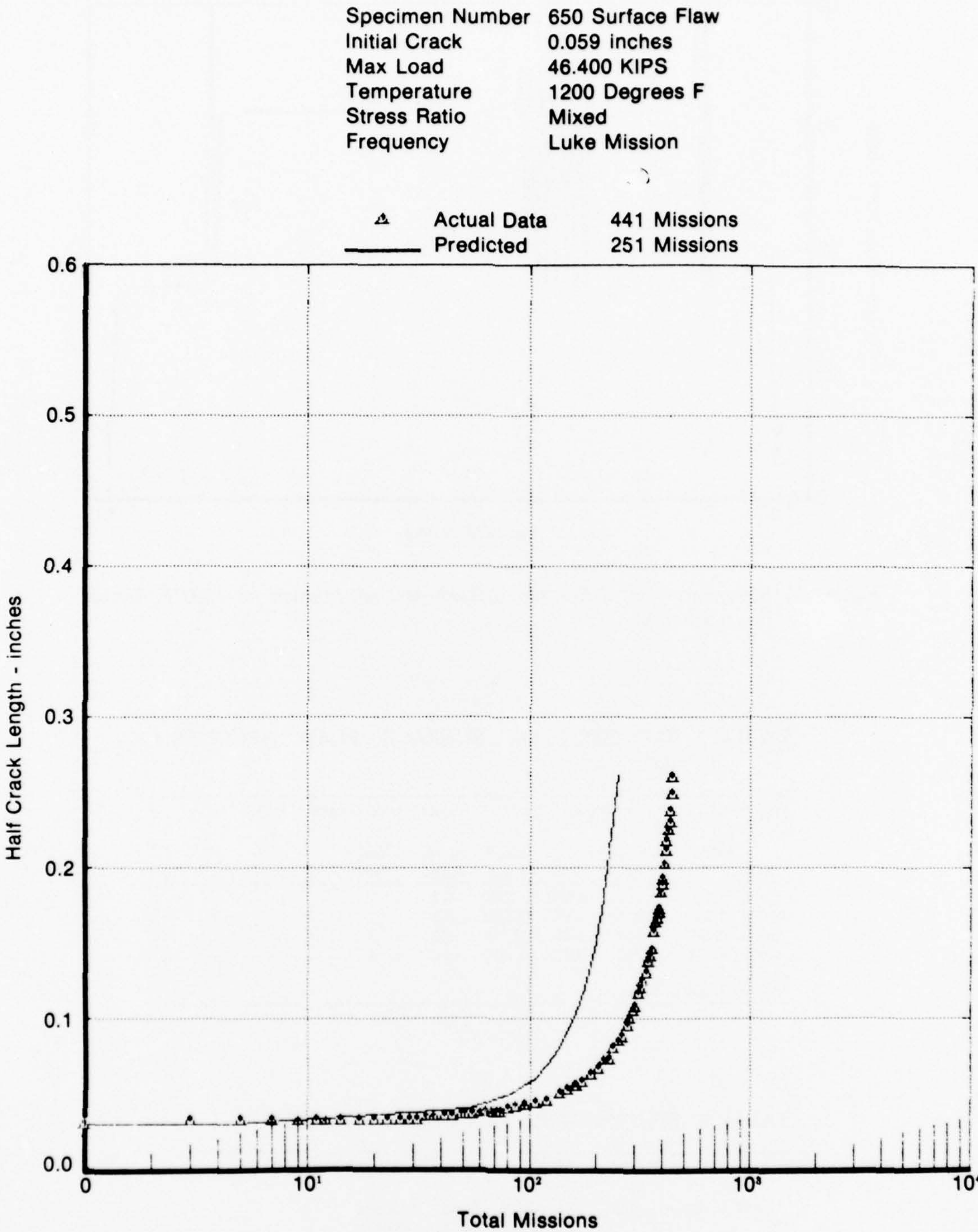


Figure 18. Surface Flaw Specimen Crack Growth Prediction, Interpolative Hyperbolic Sine Model, No Synergistic Interaction, Linear Superposition

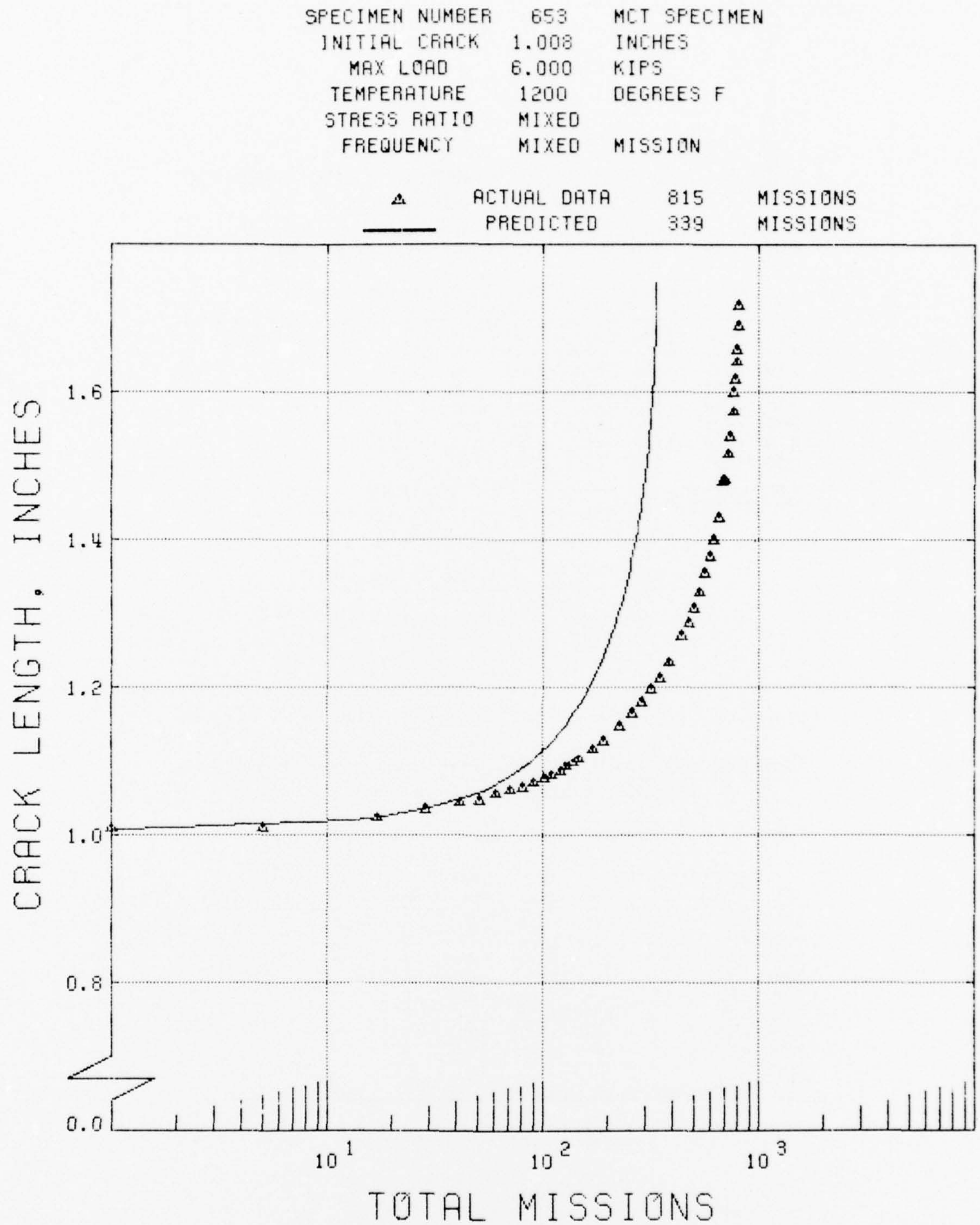


Figure 19. Modified Compact Tension Specimen Predicted Crack Growth, Interpolative Hyperbolic Sine Model, No Synergistic Interaction, Linear Superposition

TABLE 4. SPECIMEN 650 SURFACE FLAW SPECIMEN 1200°F, LINEAR SUPERPOSITION

Width = 2.509 Thickness = 0.851 Initial Crack Length = 1.0083

SINH Coefficients				*Max**	*Stress*	*Mission*
C1	*C2*	*C3*	*C4*	*Load*	*Ratio*	*Part*
0.500	3.938	−1.397	−4.156	2.760	0.20	1
0.500	4.297	−1.479	−2.519	2.760	1.00	2
0.500	4.116	−1.305	−4.241	6.000	0.46	3
0.500	4.297	−1.479	−2.519	4.680	1.00	4
0.500	4.169	−1.277	−4.267	5.340	0.52	5
0.500	4.297	−1.479	−2.519	4.680	1.00	6
0.500	4.116	−1.305	−4.241	6.000	0.46	7
0.500	4.297	−1.479	−2.519	2.760	1.00	8

*All loads are in Kips.

TABLE 5. SPECIMEN 653 MCT SPECIMEN 1200°F, LINEAR SUPERPOSITION

Width = 1.171 Thickness = 0.449 Initial Crack Length = 0.02950

SINH Coefficients				*Max**	*Stress*	*Mission*
C1	*C2*	*C3*	*C4*	*Load*	*Ratio*	*Part*
0.500	3.938	−1.397	−4.156	21.344	0.20	1
0.500	4.297	−1.479	−2.519	21.344	1.00	2
0.500	4.116	−1.305	−4.241	46.400	0.46	3
0.500	4.297	−1.479	−2.519	36.192	1.00	4
0.500	4.169	−1.277	−4.267	41.296	0.52	5
0.500	4.297	−1.479	−2.519	36.192	1.00	6
0.500	4.116	−1.305	−4.241	46.400	0.46	7
0.500	4.297	−1.479	−2.519	21.344	1.00	8

*All loads are in Kips.

TABLE 6. SPECIMEN 650 SURFACE FLAW SPECIMEN 1200°F, RETARDATION

Width = 1.171   Thickness = 0.449   Initial Crack Length = 0.0295

SINH Coefficients				*Max**	*Stress*	*Mission*
C1	*C2*	*C3*	*C4*	*Load*	*Ratio*	*Part*
0.500	3.938	−1.397	−4.156	21.344	0.20	1
0.500	4.297	−1.479	−2.519	21.344	1.00	2
0.500	4.116	−1.305	−4.241	46.400	0.46	3
0.500	4.378	−1.633	−2.998	36.192	1.00	4
0.500	4.297	−1.479	−2.519	36.192	1.00	5
0.500	4.169	−1.277	−4.267	41.296	0.52	6
0.500	4.292	−1.565	−2.807	36.192	1.00	7
0.500	4.297	−1.479	−2.519	36.192	1.00	8
0.500	4.116	−1.305	−4.241	46.400	0.46	9
0.500	4.297	−1.479	−2.519	21.344	1.00	10

*All loads are in Kips.

TABLE 7. SPECIMEN 653 MCT SPECIMEN 1200°F, RETARDATION

Width = 2.509   Thickness = 0.851   Initial Crack Length = 1.0083

SINH Coefficients				*Max**	*Stress*	*Mission*
C1	*C2*	*C3*	*C4*	*Load*	*Ratio*	*Part*
0.500	3.938	−1.397	−4.156	2.760	0.20	1
0.500	4.297	−1.479	−2.519	2.760	1.00	2
0.500	4.116	−1.305	−4.241	6.000	0.46	3
0.500	4.378	−1.633	−2.998	4.680	1.00	4
0.500	4.297	−1.479	−2.519	4.680	1.00	5
0.500	4.169	−1.277	−4.267	5.340	0.52	6
0.500	4.292	−1.565	−2.807	4.680	1.00	7
0.500	4.297	−1.479	−2.519	4.680	1.00	8
0.500	4.116	−1.305	−4.241	6.000	0.46	9
0.500	4.297	−1.479	−2.519	2.760	1.00	10

*All loads are in Kips.

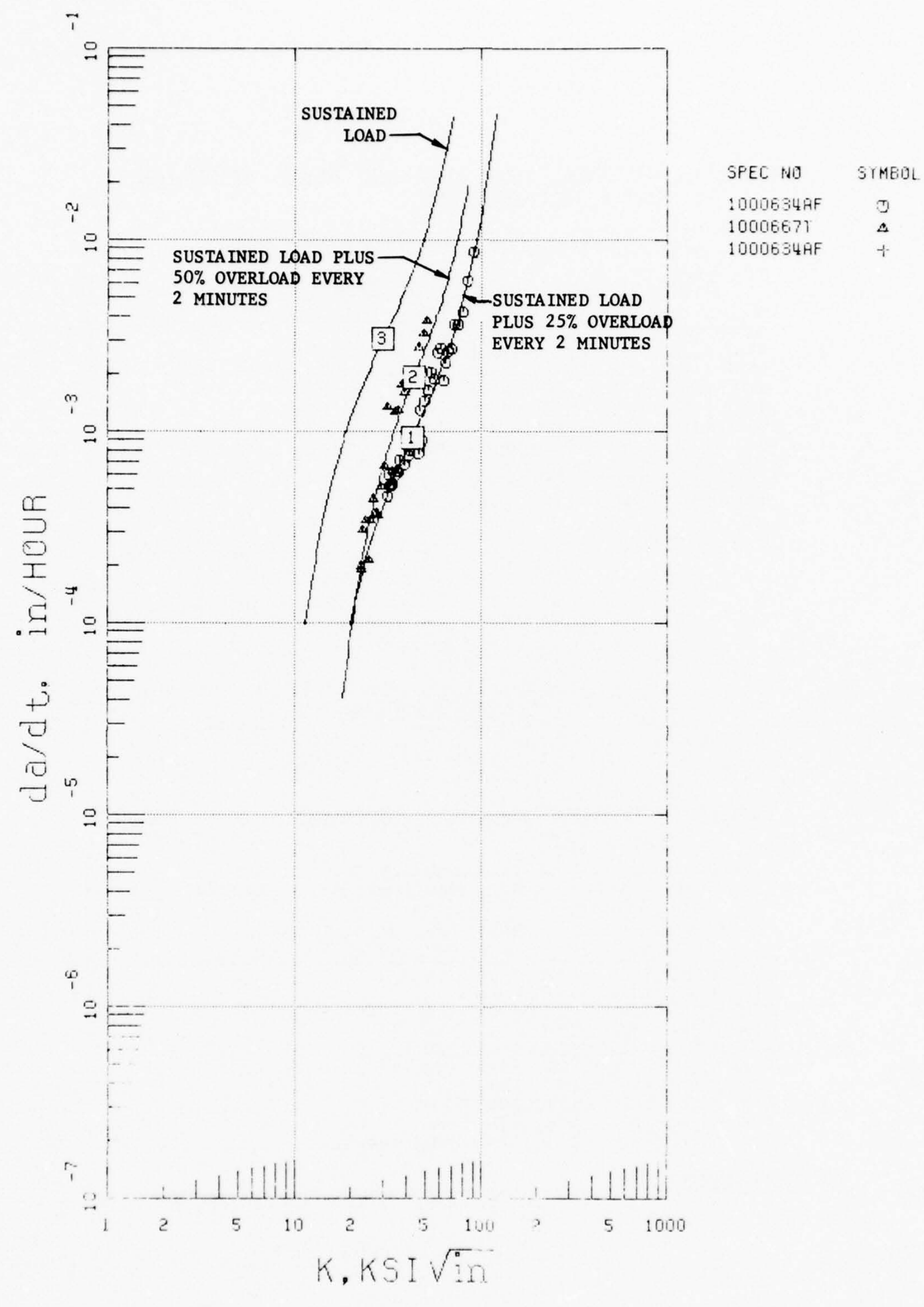


Figure 20. Effect of 25% and 50% Multiple Overloads on Sustained Load Crack Growth in IN-100, 1200°F

The following equations detail the relationships between overload ratio (OLR) and the resulting SINH coefficients which model crack growth rate.

The model used to determine synergistic SINH equation coefficients for mission parts 4, 7 and 10 (Figure 9) is defined as follows:

For $1.0 \leq OLR \leq 1.25$

$$C_1 = 0.5$$
$$C_2 = -0.036\,(OLR) + 4.333$$
$$C_3 = -0.612\,(OLR) - 0.867$$
$$C_4 = -2.056\,(OLR) - 0.463$$

For $1.25 \leq OLR \leq 1.5$

$$C_1 = 0.5$$
$$C_2 = 3.016\,(OLR) + 0.518$$
$$C_3 = -0.04\,(OLR) - 1.582$$
$$C_4 = 1.25\,(OLR) - 4.598$$

The SINH equation coefficients for the other mission parts are for exact conditions using the interpolative SINH model developed in Reference 1.

The retardation prediction for the surface flaw specimen is very accurate ($N_{pred}/N_{actual} = 0.98$) as shown in Figure 21. The actual data has the shape predicted, indicating an accurate K-solution and an accurate predictive synergistic model.

The MCT specimen synergistic prediction was more accurate ($N_{pred}/N_{actual} = 0.69$) than the linear superposition prediction, but it was still conservative (Figure 22). The shape of the actual a vs N data is slightly different from the prediction. A check was made to determine if this discrepancy could be due to the test load being much higher than precrack load. The higher load could have created a reduction in crack curvature causing apparent accelerated crack growth at the beginning of the test. Starting the prediction at a longer crack length, after the precrack effects have disappeared (17 missions), matches the predicted shape to the actual data in the early part of the test (Figure 23), but there is apparently more retardation at higher stress intensities than predicted by the model.

The accuracy of the MCT K-solution (as well as the computer program accuracy) was checked by predicting the life of a simple cyclically loaded specimen. An MCT specimen was tested at 1100°F, R = 0.3, at 2 cpm, and the actual da/dn vs ΔK data was fit with the hyperbolic sine equation. This regressed equation was then integrated by the life analysis program to determine if the actual a vs N data could be reproduced, indicating an accurate K-solution. Figure 24 shows the predicted data matches the actual data very well. This indicates that the inaccurate mission mix prediction for the MCT specimen was not caused by an inaccurate K-solution.

Figure 25 compares the 1000°F linear superposition surface flaw prediction with the 1200°F linear superposition surface flaw prediction and the 1200°F retardation model surface flaw prediction. Each prediction used the same load and the same starting crack lengths.

Specimen Number	650 Surface Flaw
Initial Crack	0.059 inches
Max Load	46.400 KIPS
Temperature	1200 Degrees F
Stress Ratio	Mixed
Frequency	Luke Mission

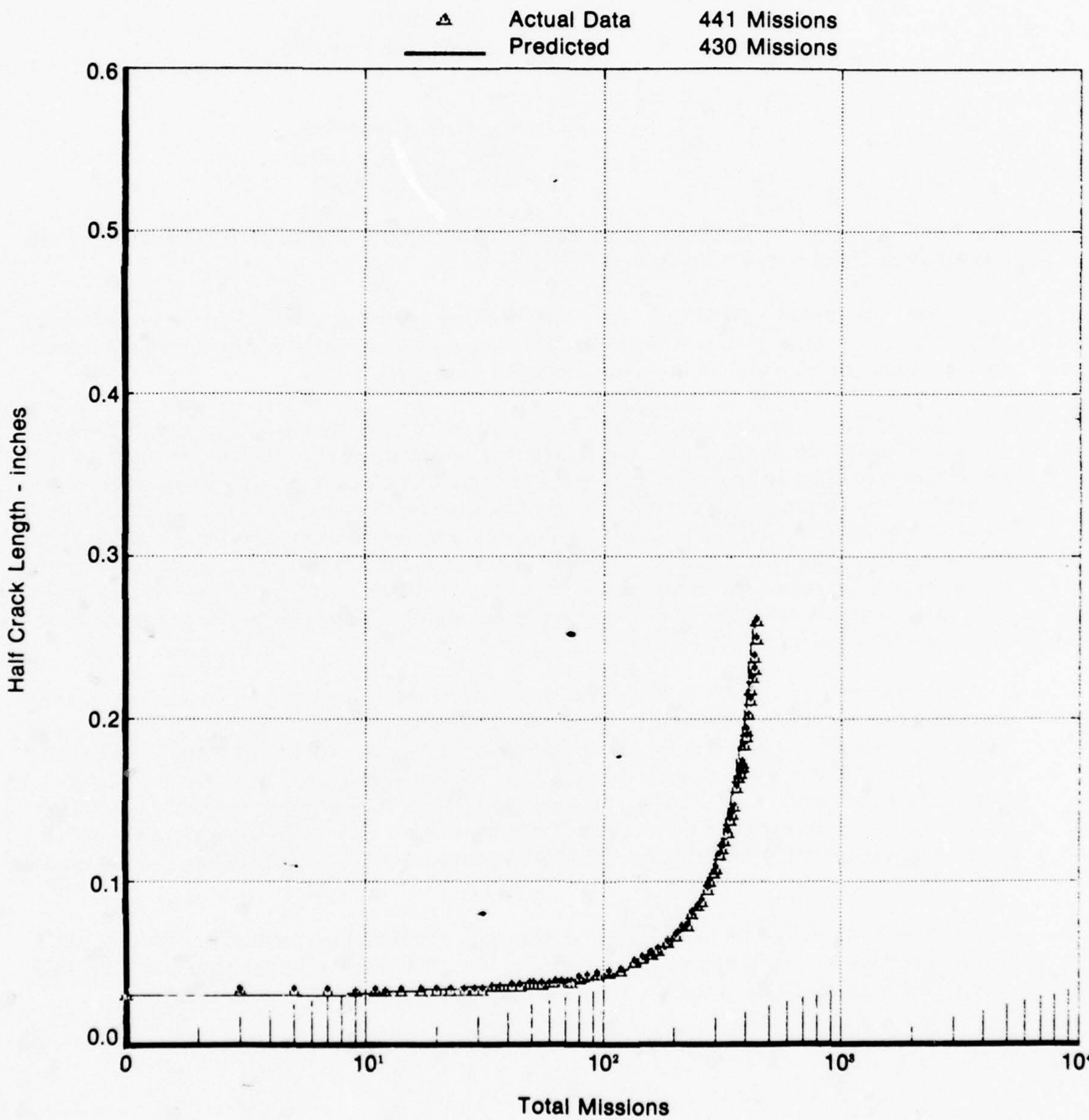


Figure 21. Surface Flaw Specimen Crack Growth Prediction, Interpolative Hyperbolic Sine Model, Retardation Model

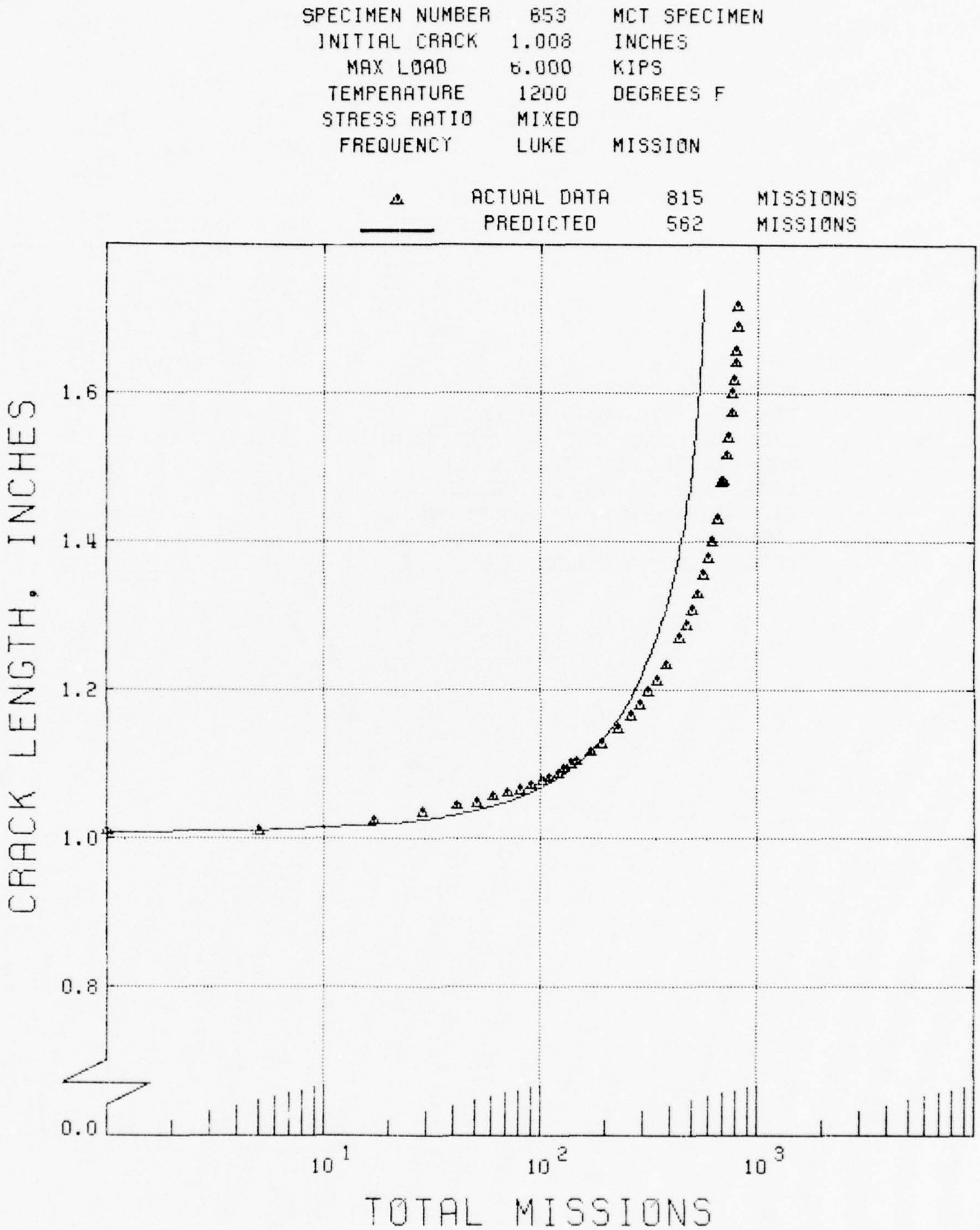


Figure 22. Modified Compact Tension Specimen Predicted Crack Growth, Interpolative Hyperbolic Sine Model, Retardation Model

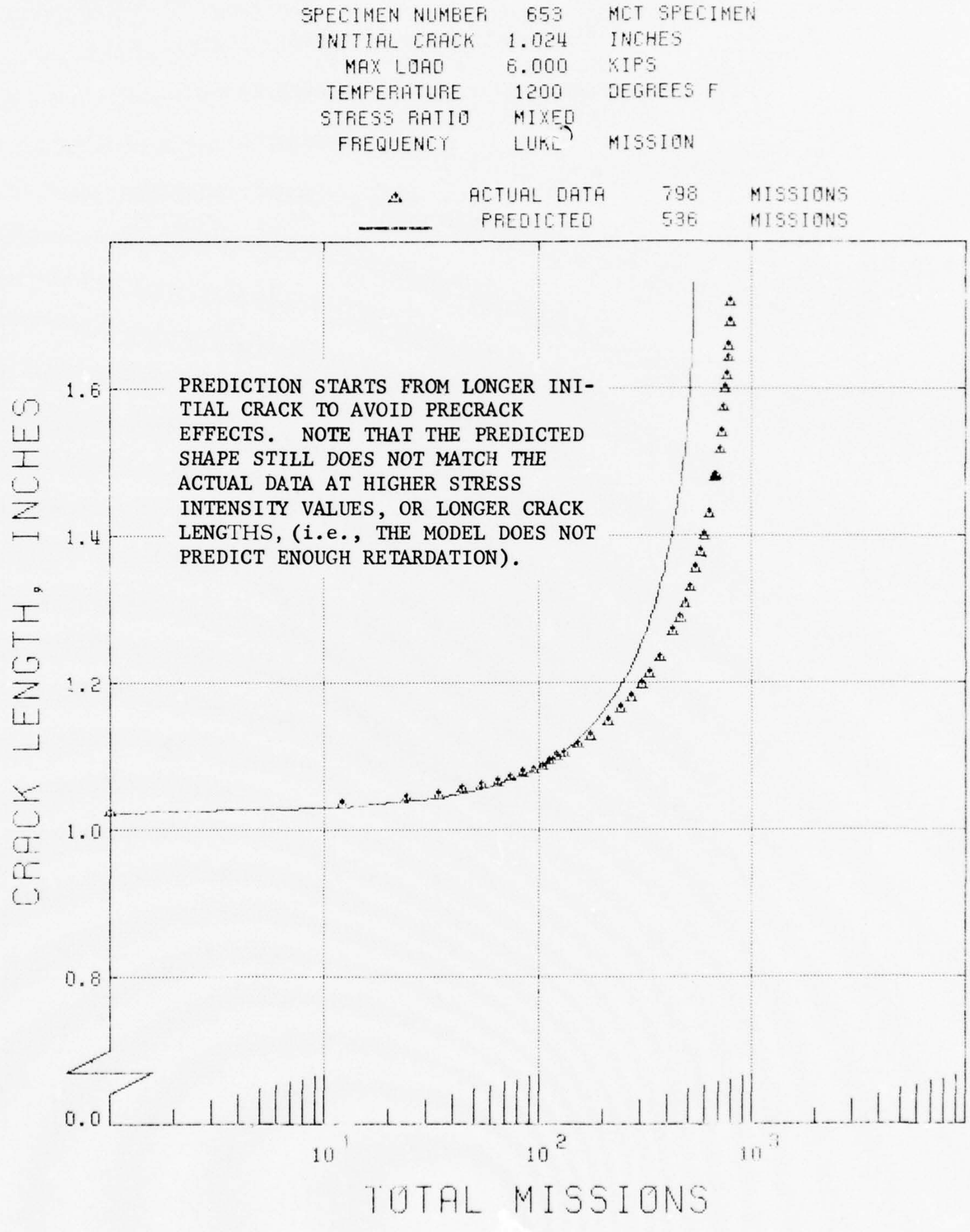


Figure 23. Modified Compact Tension Specimen Predicted Crack Growth, Interpolative Hyperbolic Sine Model, Retardation Model

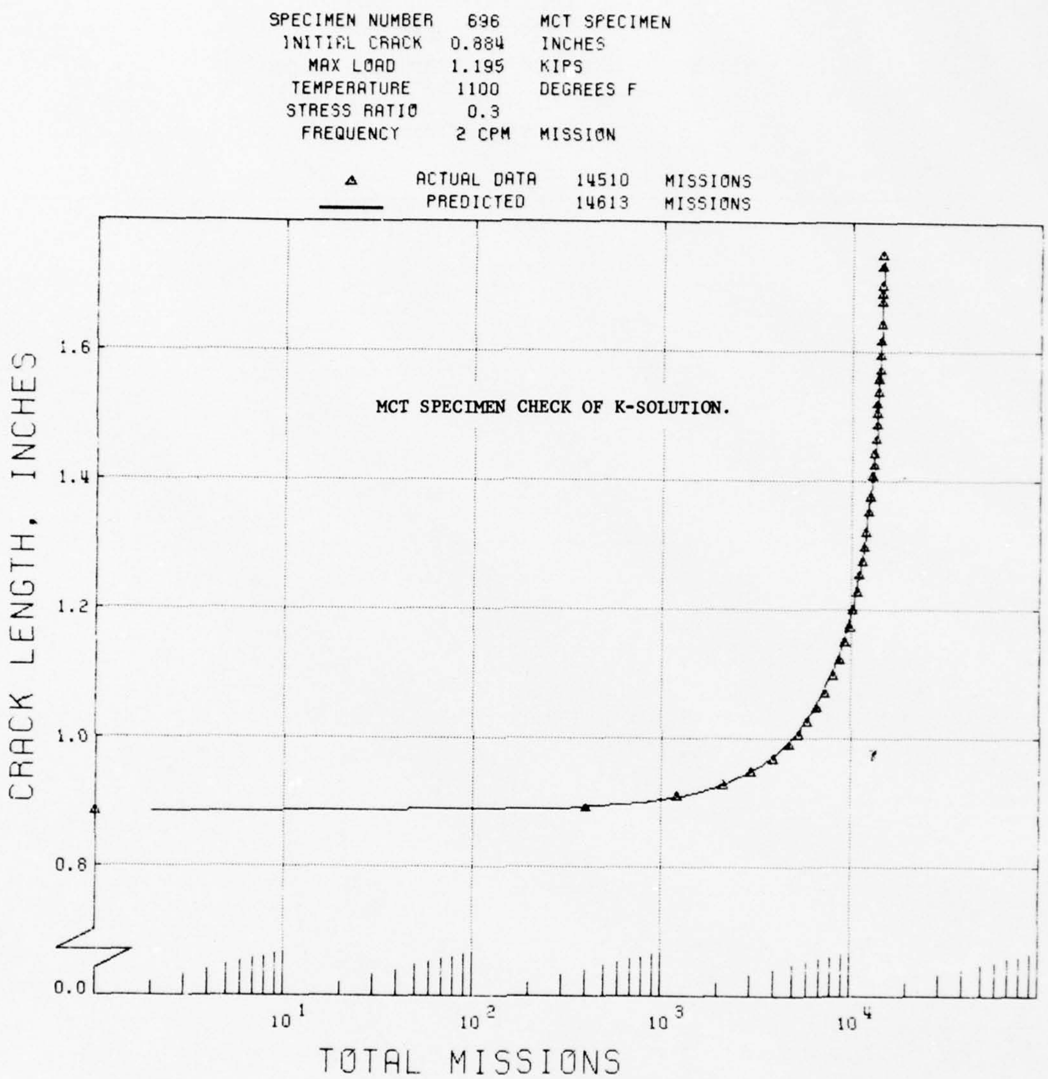


Figure 24. Modified Compact Tension Specimen Predicted Crack Growth, Interpolative Hyperbolic Sine Model, No Synergistic Interaction, Linear Superposition

Prediction	Description
1	1200°F, Linear Superposition, No Synergistic Interaction
2	1200°F, Sinh Retardation Model
3	1000°F, Linear Superposition, No Synergistic Interaction

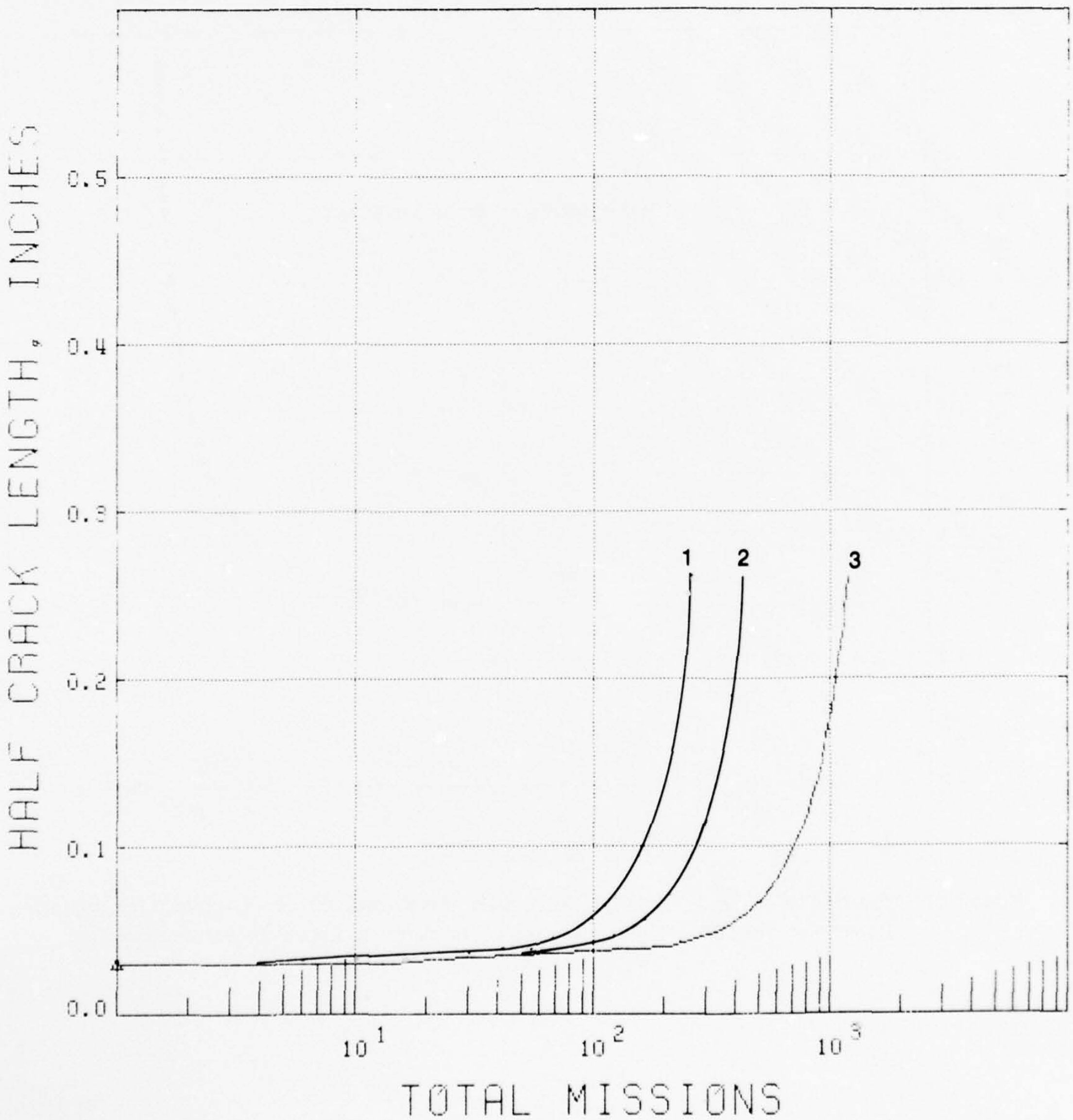


Figure 25. Comparison of 1000°F and 1200°F Linear Superposition Predictions With the 1200°F Retardation Prediction

SECTION IV
CRITIQUE

The hyperbolic sine model has been demonstrated to provide accurate life predictions for crack propagation at elevated temperatures under complex mission spectra. The accuracy of the predictions for the surface crack specimens (N_{pred}/N_{actual} = 0.91 and 0.98) was better than for the modified compact tension specimens (N_{pred}/N_{actual} = 1.12 and 0.69). This result is interesting since the bulk of the data used in model development came from MCT specimens (Reference 1).

Figures 26 and 27 are macrofractographs of the fracture surfaces of the surface-crack specimens tested at 1000°F and 1200°F, respectively. The cracks behaved well and the assumption of a semicircular crack in the life analysis is verified by the fractographs. As expected, there is some evidence of crack turn back at the free surface. The absolute accuracy of the predictions and the agreement in predicted crack history (shape of the a vs N curves relative to actual data) leads to the conclusion that both the K-solution and the empirical SINH models are accurate.

Since the same models are used for the MCT specimen life predictions, a problem must exist with test procedures. Figures 28 and 29 are macrofractographs of the fracture surfaces of the MCT specimens tested at 1000°F and 1200°F, respectively. A shear lip is observed on the specimen tested at 1000°F. During constant amplitude load testing at 1000°F, 0.250-inch specimens did not exhibit shear lips up to $\Delta K = 50$ KSI $\sqrt{\text{in}}$. The shear lip observed on this 0.85-in. specimen may therefore be an unexpected product of the major load excursions. Additionally, the lower temperatures create crack tip environments that are more conducive to mix mode conditions (Reference 6). The 1000°F model was developed with thinner ($\leq$ 0.5 inch) specimen data and might be mixed mode (slower da/dN). The thicker demonstration specimen might be expected to exhibit a higher crack propagation rate because of increased plane strain constraint.

The specimen tested at 1200°F has considerable crack front curvature (10%). Crack curvature affects the accuracy of the K-solution and therefore the predicted crack history. This degree of curvature was not generally observed on thinner (0.25 in.) specimens subjected to mission mix cycling (Reference 6). The curvature observed here is a result of the curved precrack which resulted from room temperature (ductile) precrack cycling. Room temperature precracking in thinner specimens ($\leq$ 0.5 in.), however, produces negligible crack front curvature, so the effect here was unexpected.

The thicker MCT specimens were machined differently (removed the chevron) than the specimens used to characterize the material. This modification was to reduce curvature during precrack, but curvature was still excessive.

The empirical SINH model has been demonstrated to be an effective vehicle for interpolative life prediction under both simple and complex loading spectra. Its strength lies in its empirical description of observed material behavior (crack propagation) rather than any quasianalytical model of crack tip deformation and hypothesized subsequent effects on crack advance.

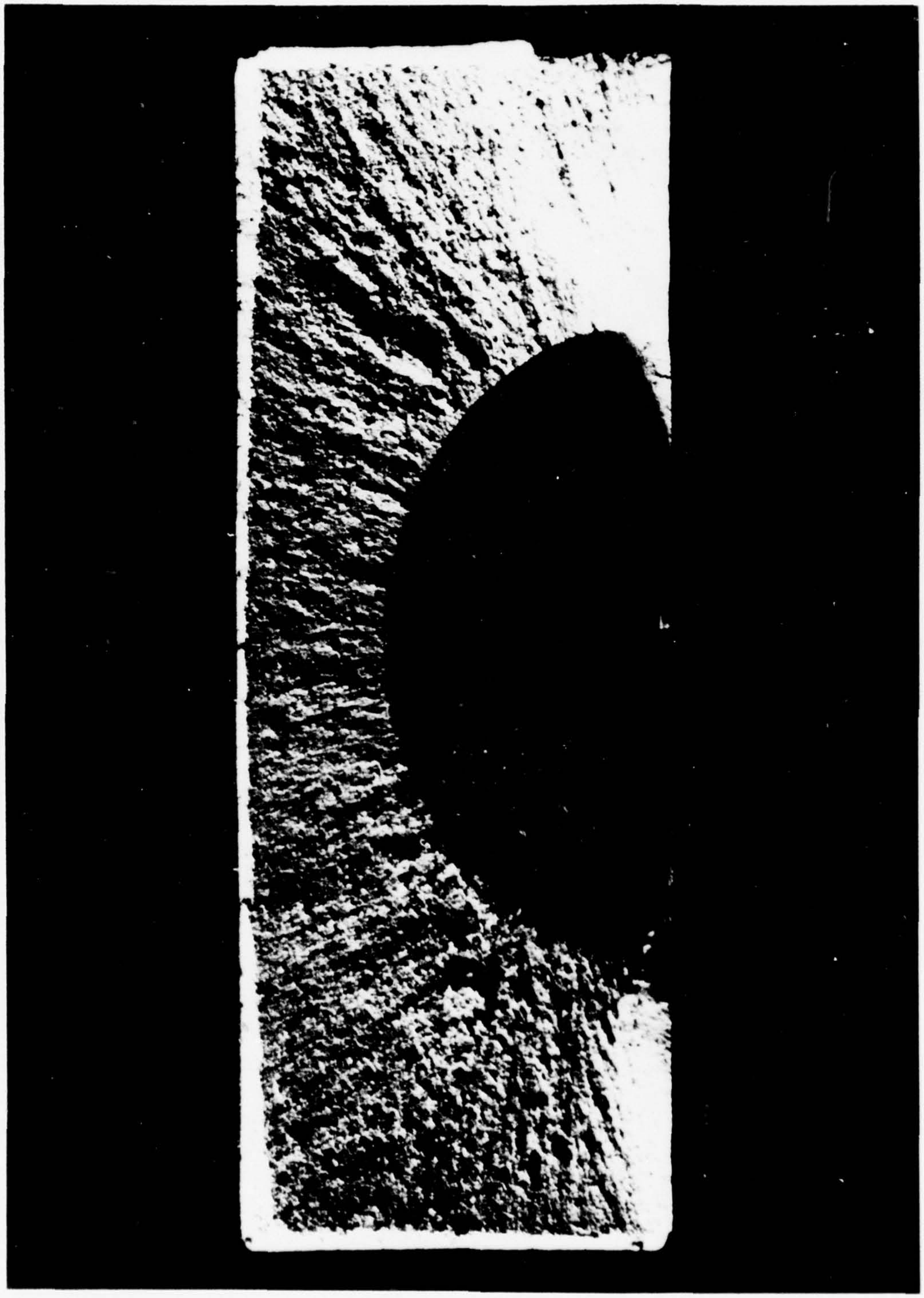


Figure 26. Macrofractograph for 1000°F Surface Flaw Specimen

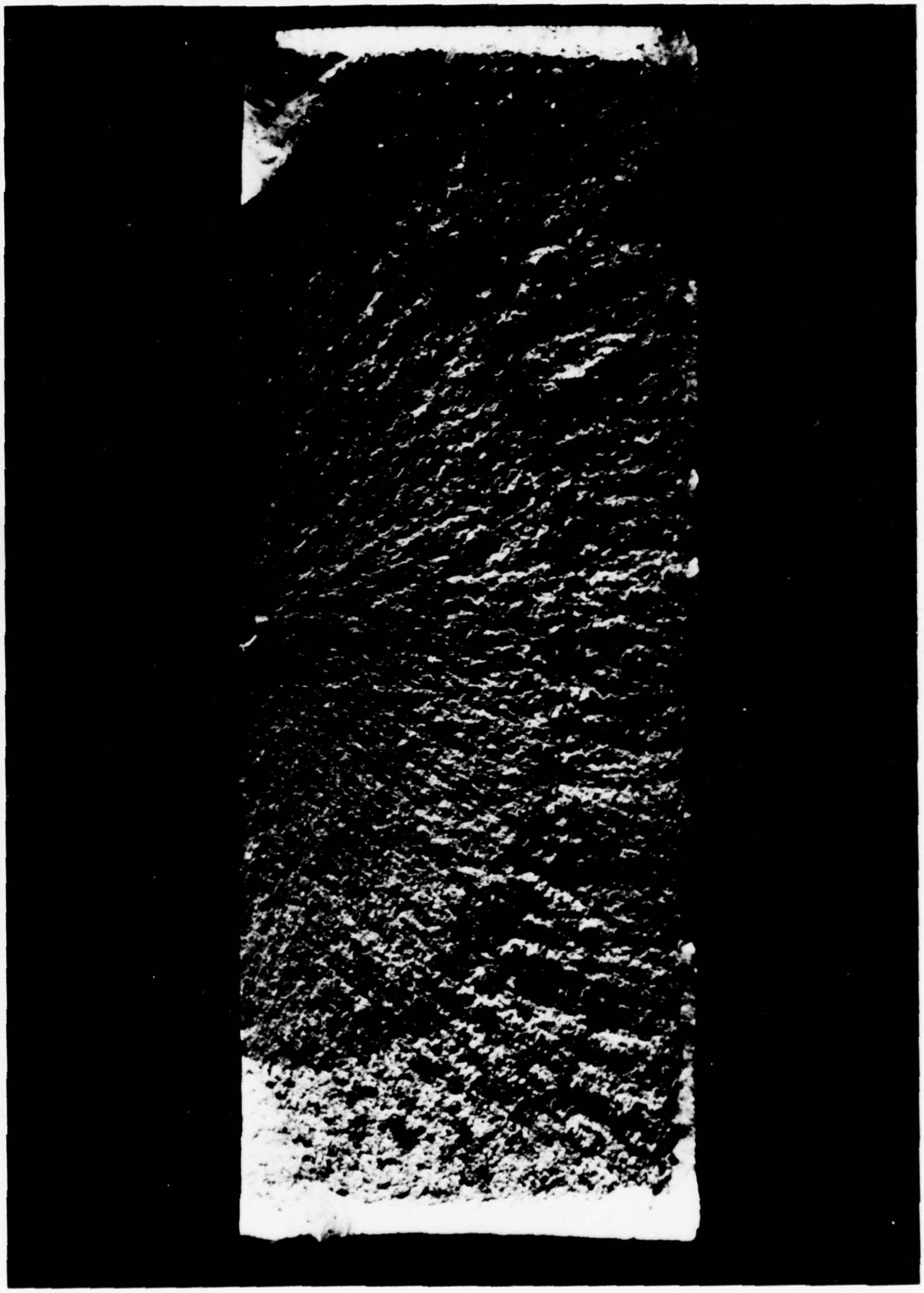


Figure 27. Macrofractograph for 1200°F Surface Flaw Specimen

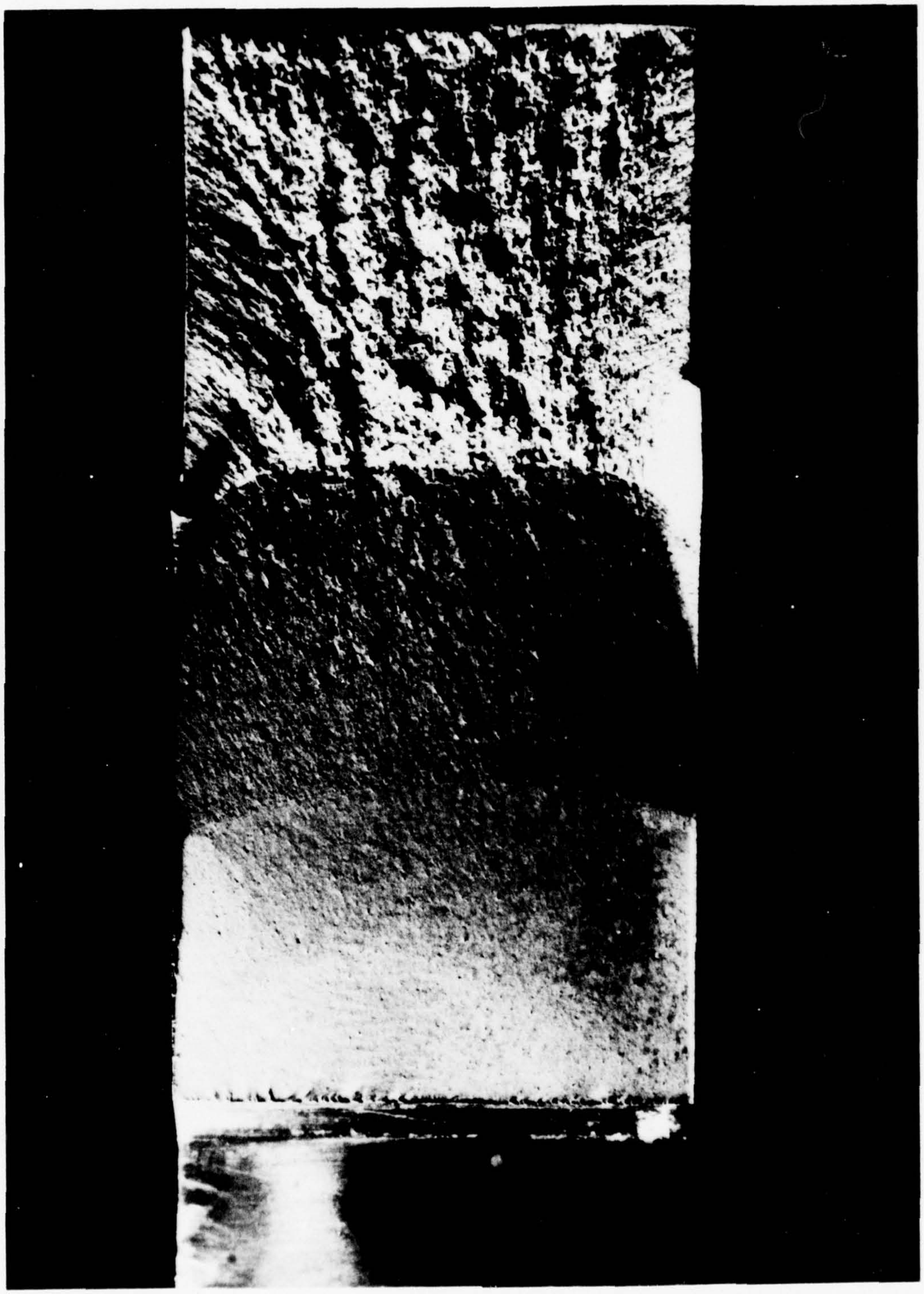


Figure 28. Macrofractograph for 1000°F MCT Specimen

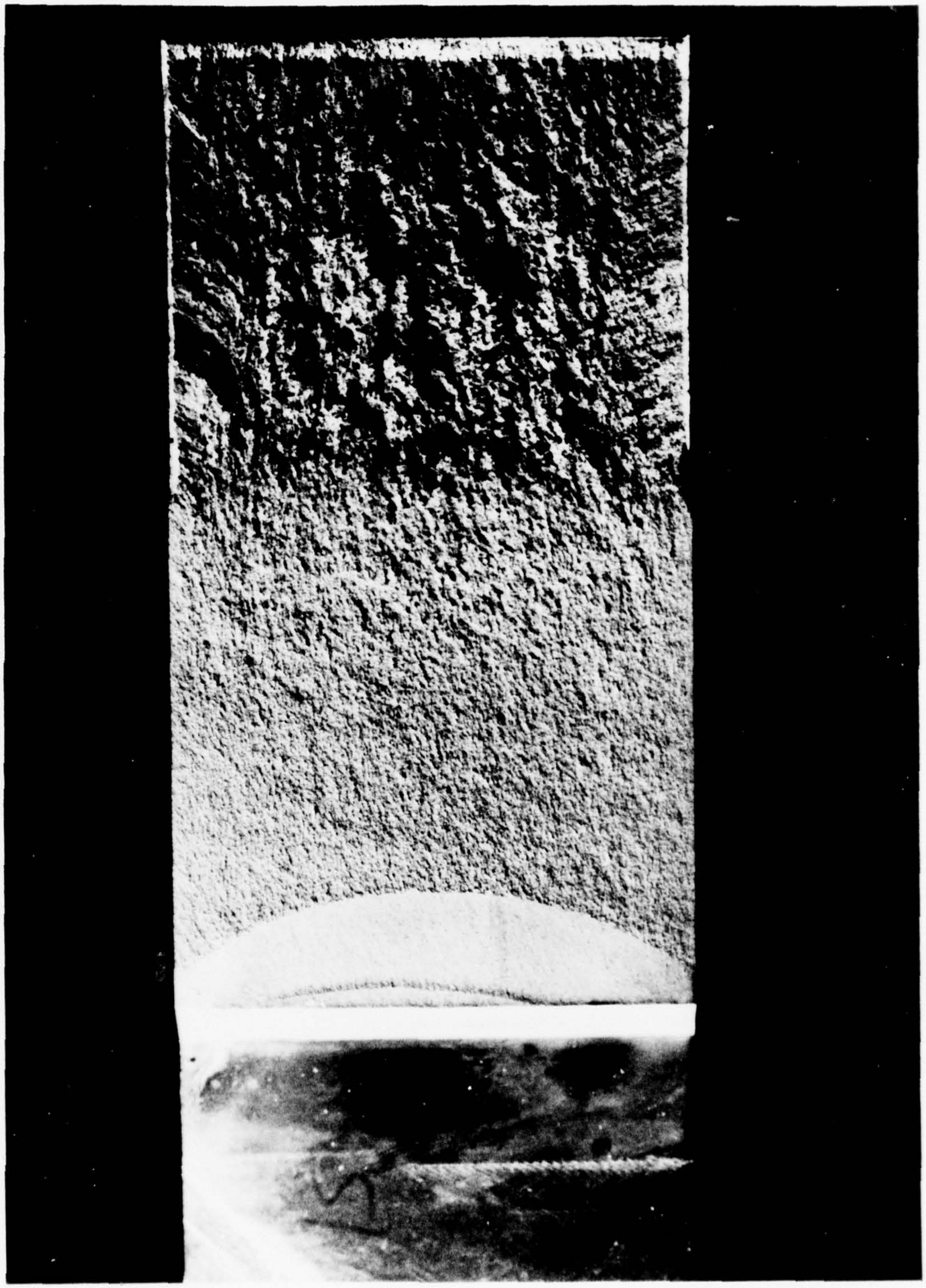


Figure 29. Macrofractograph for 1200°F MCT Specimen

SECTION V
CONCLUSIONS

1. The interpolative hyperbolic sine model accurately predicts crack propagation life at elevated temperatures under complex mission spectra.

2. Linear superposition techniques are applicable in IN-100 at temperatures where environmental degradation is minimal (1000°F).

3. Retardation and acceleration in crack growth of IN-100 due to major load excursions should be accounted for at temperatures in the creep regime (1200°F).

4. Further study is required to determine if empirical synergistic models are generally applicable to different component geometries and crack geometries.

5. Lack of time prevented running all four demonstration tests (2 at 1000°F and 2 at 1200°F) at the same load. Table 8 gives a list of starting stress intensity values. A hypothetical comparison is presented in Figure 25.

TABLE 8. STARTING STRESS INTENSITIES

Specimen	*Type*	*Temperature °F*	*Starting K ksi Inch*
650	SF	1200	14
651	SF	1000	15
652	MCT	1000	50
653	MCT	1200	30

REFERENCES

1. Annis, C. G., R. M. Wallace, D. L. Sims, "An Interpolative Model for Elevated Temperature Fatigue Crack Propagation," Air Force Materials Laboratory, AFML-TR-76-176, November 1976.

2. Schijve, J., "Observations on the Prediction of Fatigue Crack Growth Propagation Under Variable-Amplitude Loading," *Fatigue Crack Growth Under Spectrum Loads*, ASTM STP 595, pp. 3-23, 1976.

3. James, L. A., "The Effect of Frequency Upon The Fatigue-Crack Growth of Type 304 Stainless Steel at 1000°F," *Proceedings of 1971 National Symposium on Fracture Mechanics*, Part I, ASTM STP 513, pp. 218-229, 1972.

4. Meyers, G. J., "Design & K-Calibration of Surface Flaw Test Specimen," Pratt & Whitney Aircraft, Commercial Products Division Memo, June 22, 1976.

5. Shah, R. C. and A. S. Kobayashi, "On The Surface Flaw Problem," *The Surface Crack: Physical Problems and Computational Solutions*, New York: The American Society of Mechanical Engineers, 1972.

6. Wallace, R. M., C. G. Annis, and D. L. Sims, "Application of Fracture Mechanics at Elevated Temperatures," Air Force Materials Laboratory AFML-TR-76-176 Part II, November 1976.

7. Wallace, R. M., C. G. Annis, D. L. Sims, "Application of Fracture Mechanics at Elevated Temperatures," Pratt & Whitney Aircraft/Florida FP-7506, March 1976.

APPENDIX A
LIFE ANALYSIS COMPUTER PROGRAM USER INSTRUCTION

This program will predict crack propagation life for MCT specimens and surface flaw specimens with semicircular flaws. To predict crack propagation life of another type specimen, the stress intensity solution would have to replace that of the MCT specimen (if initial crack length is greater than 0.5 in.) or the surface flaw specimen (if initial crack length is less than 0.5 in.). Subroutine life contains the stress intensity solutions and the hyperbolic sine equation that models crack propagation. The sinh equation is:

$$\log (da/dn) = C_1 \sinh \left(C_2 (\log (\Delta K) + C_3) \right) + C_4 \qquad \text{(A-1)}$$

The input data cards must be in order as follows:

First Card: Card columns 1-72 are for the title. This is in "A" format under the variable name of TITL.

Second Card: Card columns 1-8 are for the frequency. The designation for frequency (FREQ) should be centered in the first eight card columns for output on the GOULD plot. FREQ is in "A" format.

Third Card: Card columns 1-8 are for listing the stress ratio for GOULD plot output. The designation for stress ratio (RR) should be centered in the first eight columns. RR is in "A" format.

Fourth Card: Card columns 1-52 can be used for additional title such as type of analysis. Whatever is input should be centered in the 52 columns in "A" format.

Fifth Card: Beginning with this card, the NAMELIST parameters are input in the format shown in the sample data set (Appendix C). The input begins with &INPUT, then a blank space. A listing of the parameters and their values is followed by &END which ends the NAMELIST input. Commas separate each parameter value. Table A-1 defines each NAMELIST variable.

Sixth Card: The actual data is listed next. A maximum of 100 data points may be input. Card columns 1-10 are for the crack length (half crack length for surface flaw specimens) and columns 11-20 are for the number of missions corresponding to the crack length (this cannot be zero). One data point must always be input, the initial crack length and one mission. Both variables use "F" format.

Seventh Card: A -1.0 is input in card columns 1-10 to separate data sets and allow multiple case analysis.

TABLE A-1
NAMELIST VARIABLES

PERIOD:	Normally PERIOD is equal to one for a cyclic portion of a mission, but *must be less than one* (units in hours) for sustained load portions of a mission. For better accuracy during sustained load portions of a misssion, PERIOD should be less than 0.1 hours. To shorten PERIOD, increase NPER (PERIOD * NPER = total length of mission part).
NPER:	Number of cycles or time increments in a mission part.
COEF1:	C_1 in the hyperbolic sine crack propagation equation A-1.
COEF2:	C_2 in the hyperbolic sine equation A-1.
COEF3:	C_3 in the hyperbolic sine equation A-1.
COEF4:	C_4 in the hyperbolic sine equation A-1.
PMAX:	Maximum load for each part of the mission.
RATIO:	Stress Ratio for each part of the mission.
AI:	Initial Crack length.
AF:	Final Crack length at failure (or at the end of stress intensity solution validity).
TEMP:	The temperature to be printed on the GOULD plot.
PKLD:	The maximum load for the whole mission. This is used only for GOULD plot output.
WID:	The width of the surface flaw specimen gage section, or the distance from the MCT load centerline to the end of the specimen.
THIK:	Thickness of the specimen.
LOOPS:	The number of parts into which the mission has been divided.

APPENDIX B
ACTUAL PROGRAM

```
RELEASE 2.0                 MAIN                DATE = 76314          16/32/20

C      LIPHE.FORT ---- WRITTEN BY D.L. SIMS AND C.G. ANNIS.  LATEST
C      REVISION 10/29/76.  THIS PROGRAM PREDICTS THE LIFE OF MIXED
C      MISSON SPECIMENS FOR PHASE III OF THE AFML FRACTURE MECHANICS
C      CONTRACT (MCT AND SURFACE FLAW SPECIMENS ONLY).
C
C
C
C      EXAMPLE DATA CARDS ARE AS FOLLOWS
C   1  CC-1 TO 72 IS FOR THE TITLE.
C
C   2  CC-1 TO 8 IS FOR THE FREQUENCY FOR GOULD PLOT OUTPUT ONLY.
C
C   3  CC-1 TO 8 IS FOR THE STRESS RATIO FOR GOULD PLOT OUTPUT ONLY.
C
C   4  CC-1 TO 52 IS FOR THE TYPE LIFE PREDICTION (SUPERPOSITION, ETC).
C
C   5  NEXT COMES NAMELIST DATA (SEVERAL CARDS).
C
C   6  THE ACTUAL A,N DATA IS LAST (1-100 CARDS).
C
C   7  -1.0 MUST BE PLACED BETWEEN DIFFERENT CASES.
C
       DIMENSION A(100), XN(100), CA(100), CN(100), COEF1(20), COEF2(20),
     1 COEF3(20), COEF4(20), TITL(18), NPER(20), PERIOD(20),CARD(20),
     2 PMAX(20), FREQ(2), RATIO(20), RR(2), TYPE(13)
       DATA A, XN, CA, CN/ 400*0./
       DATA COEF1, COEF2, COEF3, COEF4, PERIOD, PMAX, NPER/120*0.,20*0/
       DATA RATIO/20*0./
C
       NAMELIST  /INPUT/COEF1,COEF2,COEF3,COEF4,AI,AF,TEMP,PMAX,
     1             WID,THIK,RATIO,NPER,PERIOD,PKLD,LOOPS
C
C
C      WID=WIDTH, THIK=THICKNESS, RATIO=STRESS RATIO, TITL=CC1-72 OF
C      TITLE CARD, XN=ACTUAL LIFE, COEF1-COEF2-COEF3-COEF4=SINH COEF,
C      AI=INITIAL CRACK LENGTH, AF=CRITICAL CRACK LENGTH, DA=DELTA CRACK
C      LENGTH, PMAX=MAX LOAD, A=ACTUAL CRACK LENGTH, CA=CALCULATAED CRACK
C      LENGTH STORED, TA=CALCULATAED CRACK LENGTH, CN=CALCULATED LIFE
C      STORED, TN=CALCULATED LIFE, DKONE=K MAX, DK=DELTA K, PKLD=PEAK
C      LOAD, FREQ=FREQUENCY, MISN = NUMBER OF MISSIONS, RR = STRESS RATIO
C      FOR GOULD PRINTOUT ONLY, PERIOD = TIME OR CYCLES IN A MISSION
C      PART, LOOPS = NUMBER OF PARTS IN THE MISSION, NPER = NUMBER OF
C      CYCLES OR TIME INCREMENTS IN A MISSION PART.  TYPE = TYPE
C      OF LIFE PREDICTION (SUPERPOSITION, ETC).
C
C      READ AND WRITE THE COMPLETE DATA SET.
       WRITE(6,59)
    59 FORMAT(1H1)
    51 READ(5,53,END=55)CARD
    53 FORMAT(20A4)
       WRITE(6,57)CARD
    57 FORMAT(1X,20A4)
       GO TO 51
    55 REWIND 5
       WRITE(6,59)
C
    10 CONTINUE
C
```

```
C     INITIALIZE.
      DA=0.
      TN=0.
      NPTS = 0
      MOUNT = 0
      LESTA = 0
      INT = 0
      INCT = 1
C
C
C     READ AND WRITE THE INDIVIDUAL PARAMETERS.
      READ(5,1,END=999)TITL
    1 FORMAT(18A4)
      WRITE(6,59)
      WRITE(6,194) TITL
  194 FORMAT(//1X,18A4/)
C
C     READ THE FREQUENCY AND STRESS RATIO. FOR OUTPUT ON THE GOULD ONLY.
      READ(5,4,END=999)FREQ, RR, TYPE
    4 FORMAT(2A4/2A4/13A4)
C
C     READ THE NAMELIST PARAMETERS.
      READ(5,INPUT,END=999)
C
C     READ AND WRITE THE ACTUAL A VS N DATA.
      WRITE(6,39)
   39 FORMAT(//1X,'ACTUAL  A   VS   N   DATA')
      DO 9 J=1,100
      READ(5,93) A(J), XN(J)
   93 FORMAT(2F10.5)
      WRITE(6,92) A(J), XN(J)
   92 FORMAT(1X,F10.7,4X,F10.1)
      IF(A(J) .LE. 0.) GO TO 20
      NPTS = NPTS + 1
    9 CONTINUE
C
C     WRITE SOME OF THE NAMELIST PARAMETERS.
   20 WRITE(6,59)
      WRITE(6,203)WID,THIK,AI
  203 FORMAT(//1X,'WIDTH =',F8.5,6X,'THICKNESS =',F8.5,6X,
     1'INITIAL CRACK LENGTH =',F8.5)
C
C     WRITE THE SINH COEFFICIENTS, MAX LOAD AND STRESS RATIO FOR EACH
C     MISSION PART.
      WRITE(6,761)
  761 FORMAT(//14X,'SINH COEFFICIENTS',15X,'MAX',6X,'STRESS',
     13X,'MISSION')
      WRITE(6,762)
  762 FORMAT(/6X,'C1',8X,'C2',8X,'C3',8X,'C4',8X,'LOAD',6X,'RATIO',6X,
     1'PART')
      DO 8 J=1,LOOPS
  492 WRITE(6,200)COEF1(J), COEF2(J), COEF3(J), COEF4(J), PMAX(J),
     1 RATIO(J), J
  200 FORMAT(//1X,4F10.5,3X,F8.5,3X,F6.3,6X,I3)
    8 CONTINUE
      WRITE(6,269) LOOPS
  269 FORMAT(//1X,'THE MISSION IS BROKEN DOWN INTO',I5,3X,'PARTS')
C
```

```
C     INITIALIZE.
      TA=AI
      MISN=0
C
C     CALCULATE THE LIFE - CALL SUBROUTINE LIFE.
C     SUBROUTINE LIFE DOES THE INTEGRATION CYCLE BY CYCLE.
C
  101 NPART = 1
      DO 155 I=1,LOOPS
C**** FIRST PART OF MISSION - 1ST  CALL TO LIFE.  ********************
      CALL LIFE(COEF1(NPART), COEF2(NPART), COEF3(NPART), COEF4(NPART),
     1   AF, TA, TN, WID, THIK, RATIO(NPART), PMAX(NPART), CA, CN,
     2   NPER(NPART),NPART,PERIOD(NPART),MOUNT,MISN,LESTA,INT,INCT,TITL,
     3   AI)
C
C     CHECK IF CRACK IS PAST THE CRITICAL LENGTH.
      IF(TA .GE. AF) GO TO 19
C
  155 CONTINUE
C
       IF(TA .LT. AF) GO TO 101
C
C ...
C     CALCULATE PREDICTED LIFE OVER ACTUAL LIFE.
   19 Q = MISN/XN(NPTS)
C
C     WRITE OUT THE RESULTS.
      WRITE(6,413) TITL
  413 FORMAT(1H1,1X,18A4)
      WRITE(6,108) TA, TN
  108 FORMAT(1X,'FINAL CRACK LENGTH =',F8.5,5X,'CYCLES TO FAILURE ='
     1,F10.1,//)
      WRITE(6,210) XN(NPTS), MISN, Q
  210 FORMAT(//1X,'ACTUAL LIFE = ',F7.0,10X,'PREDICTED LIFE = ',
     1I10//1X,'PREDICTED/ACTUAL = ',F8.5//)
C
C     SUBROUTINE CMPAR PLOTS THE PREDICTED VS ACTUAL DATA.
      CALL CMPAR(CA, CN, A, XN, NPTS, MOUNT, TITL(3), AI, PKLD, TEMP,
     1 RATIO, FREQ, MISN, XN(NPTS), RR, TYPE)
C
      GO TO 10
  999 STOP
      END
```

```
      SUBROUTINE LIFE(COEF1, COEF2, COEF3, COEF4, AF,TA, TN, WID,
     1 THIK, RATIO, PMAX, CA, CN, NPER, NPART, PERIOD, MOUNT, MISN,
     2 LESTA, INT, INCT, TITL, AI)
C
C *** CALCULATE LIFE
      DATA D1,D2,D3,D4,D5/ 29.6,-185.5,655.7,-1017.,638.9/
      DATA E1,E2,E3,E4,E5/ 0.5,1.5,2.5,3.5,4.5/
      DIMENSION CA(100), CN(100), TITL(18)
C
C     DO THE CYCLE BY CYCLE INTEGRATION.
  102 DO 169 I=1,NPER
C
C     CHOOSE IF SURFACE FLAW OR MCT SPECIMEN.
      IF(AI .LT. 0.5) GO TO 1333
C
C     K-SOLN FOR MCT SPECIMEN.
      AOW=TA/WID
      F=D1*AOW**E1 +D2*AOW**E2 +D3*AOW**E3 +D4*AOW**E4 +D5*AOW**E5
      DKONE=(PMAX/(THIK*SQRT(WID)))*F
C
      GO TO 1334
C
C     K-SOLN FOR SURFACE FLAW SPECIMEN.
 1333 DKONE=1.2063*PMAX*SQRT(TA)/(WID*THIK)
C
 1334 IF(PERIOD .LT. 1.) GO TO 5
      DK=(1.-RATIO)*DKONE
C     WRITE(6,2)AOW,F,DKONE,DK
    2 FORMAT(1X,1P4E12.5)
      GO TO 8
    5 DK=DKONE
    6 DADNLG=COEF1*SINH(COEF2*(ALOG10(DK)+COEF3))+COEF4
      DADN = 10.**DADNLG
      DA = PERIOD * DADN
      TA=TA + DA
      TN=TN + 1.
  169 CONTINUE
C
C
C     COUNT ONLY THE NUMBER OF MISSIONS.
      IF(NPART .EQ. 1) MISN = MISN + 1
C
C
C     WRITE OUT ALL THE JUNK IN THE LOOP.
      IF(INCT .LT. 1) GO TO 1633
      MOUNT=MOUNT+1
      MODULO=MOD(MOUNT,50)
      CA(MOUNT) = TA
      CN(MOUNT) = MISN
      IF(MODULO .NE. 1) GO TO 835
      WRITE(6,413) TITL
  413 FORMAT(1H1,1X,18A4)
      WRITE(6,414)
  414 FORMAT(//1X,'CRACK LENGTH',3X,'MISSIONS TO A',7X,'DK',7X,
     1'DKMAX',8X,'DA/DN'//)
  835 WRITE(6,415) TA, MISN, DK, DKONE, DADN
  415 FORMAT(1X,F9.6,5X,I10,5X,1P3E12.5)
      LESTA = INT
```

```
 1633 INT = IA * 100.
      INCT = INT - LESTA
    9 NPART = NPART + 1
      RETURN
      END
```

```
      SUBROUTINE CMPAR(CA, CN, A, XN, NPTS, MOUNT, SPNO, AI, PMAX,
     1 TEMP, R, FREQ, MISN, ALIFE, RR, TYPE)
C
C.... SUBROUTINE CMPAR PLOTS THE ACTUAL VS PREDICTED DATA.
C
C
      DIMENSION CA(100),CN(100),A(100),XN(100),FREQ(2),RR(2),TYPE(13)
      DATA XCLNTH,NXCYCL,  YCLNTH,NYCYCL/1.5, 4,   1., 6/
C
      CALL PLOTS(13.,10.)
C
C.....DRAW THE 8.5 BY 11 RECTANGLE.
      CALL PLOT(0.,10.,2)
C     CALL PLOT(8.5,10.,3)
C     CALL PLOT(8.5,0.,2)
C     CALL PLOT(0.,0.,3)
C
C.....CHECKOUT SUBROUTINE GRAPH.
C
      CALL PLOT(2.,0.8,-3)
C
C     CALL DIFFERENT ROUTINE IF SPECIMEN IS A SURFACE FLAW.
      IF(AI .GT. 0.7) GO TO 26
      CALL CFGRPH(CA,CN,A,XN,NPTS,MOUNT,SPNO,AI,PMAX,TEMP,R,FREQ,MISN,
     1 ALIFE,RR,TYPE)
      GO TO 27
C
   26 CALL GRAPH(CA,CN,A,XN,NPTS,MOUNT,SPNO,AI,PMAX,TEMP,R,FREQ,MISN,
     1 ALIFE,RR,TYPE)
   27 CALL PLOT(0.,0.,999)
C
      RETURN
      END
```

```
      SUBROUTINE GRAPH(CA,CN,A,XN,NPTS,NCALC,SPNO,AI,PMAX,TEMP,R,FREQ,
     1 MISN, ALIFE,RR,TYPE)
C.....PLOTS   CRACK-LENGTH, CYCLES ( A,N ) DATA AND PREDICTION.
      DIMENSION CA(1),CN(1),A(1),XN(1),TYPE(13)
      DIMENSION XNLOG(100),FREQ(2),RR(2)
      DATA XCLNTH,NXCYCL,  YCLNTH,NYCYCL/1.5, 4,   1., 6/
C
C.....COMPUTE THE STARTING INTEGER EXPONENT OF TEN.
C
      MINN=ALOG10( XN(1) )
      MINCN=ALOG10( CN(1) )
      MINX=MIN0( MINN, MINCN )
      XMIN=MINX
C
C.....DRAW THE LIFE PREDICTION SEMI-LOG GRID.
      CALL LPGRD(XCLNTH,NXCYCL,YCLNTH,NYCYCL,MINX,TYPE)
C
C.....PRINT THE SALIENT PARAMETERS.
      CALL PRINT(SPNO,AI,PMAX,TEMP,R,FREQ,MISN,ALIFE,RR)
C
C.....TAKE LOGS, COMPUTE THE SCALE PARAMETERS, PLOT DATA AND PREDICTION.
C
C.....TAKE LOGS OF X-PARAMETER.
      DO 10 I=1,NPTS
      XNLOG(I) = ALOG10( XN(I) )
   10 CONTINUE
C
C.....PLOT THE DATA.
      XNLOG(NPTS+1)=XMIN
      XNLOG(NPTS+2)=1./XCLNTH
      A(NPTS+1)=0.6
      A(NPTS+2)=0.2/YCLNTH
      CALL LINE(XNLOG,A,NPTS,1,-1,2)
C
C.....TAKE LOGS OF X-PARAMETER.
      DO 20 I=1,NCALC
      XNLOG(I) = ALOG10( CN(I) )
   20 CONTINUE
C
C.....DRAW THE PREDICTION.
      XNLOG(NCALC+1)=XMIN
      XNLOG(NCALC+2)=1./XCLNTH
      CA(NCALC+1)=0.6
      CA(NCALC+2)=0.2/YCLNTH
      CALL LINE(XNLOG,CA,NCALC,1,0,0)
C
      RETURN
      END
```

```
      SUBROUTINE LPGRD(XCLNTH,NXCYCL,YCLNTH,NYCYCL,MINX,TYPE)
      DIMENSION TYPE(13)
C
C     PLOT AND ANNOTATE A SEMI-LOG LIFE PREDICTION GRID.
C
      DATA H,HOVR2/0.0875, 0.044/
      HTIM2=H*2
C
C      DRAW THE GRID
      CALL MTGRD2(XCLNTH,NXCYCL,YCLNTH,NYCYCL)
C
C.....DRAW THE Z-SHAPED INTERRUPTION.
      CALL PLOT(0.,0.2,3)
      CALL ERASE(1)
      CALL PLOT(0.,0.5,2)
      CALL ERASE(0)
      CALL PLOT(-0.3,0.35,2)
      CALL PLOT(+0.3,0.35,2)
      CALL PLOT(0.,0.2,2)
C
C.....WRITE FIGURE AND TITLE
      TOP=8.7
      CALL SYMBOL(3.0,(TOP+3.*H),HTIM2,'P&WA MATERIALS AND MECHANICS LAB
     1ORATORY',0.,39,1)
      CALL SYMBOL(3.0,TOP,HTIM2,'AFML CONTRACT  F33615-75-C-5097',
     1             0.,31,1)
      CALL SYMBOL(3.0,(TOP-3.*H),H,'FIGURE    ',0.,9,1)
      CALL SYMBOL(3.0,(TOP-5.*H),H,'MODIFIED COMPACT TENSION SPECIMEN PR
     1EDICTED CRACK GROWTH',0.,56,1)
      CALL SYMBOL(3.0,(TOP-7.*H),H,'INTERPOLATIVE HYPERBOLIC SINE MODEL'
     1,0.,35,1)
      CALL SYMBOL(3.0,(TOP-9.*H),H,TYPE,0.,52,1)
C
C.... ANNOTATE THE X-AXIS
      YLOC=-0.25
      DO 10 I=1,3
      XLOC=I*XCLNTH
      CALL NUMBER(XLOC,YLOC,H,10,0.,-1,1)
      CALL NUMBER((XLOC+HTIM2),(YLOC+H),H,(MINX+I),0.,-1,1)
   10 CONTINUE
C
C.....ANNOTATE THE Y-AXIS
      CALL NUMBER(-0.30,0.,H,0.,0., 1)
      DO 20 I=1,5
      YLOC=I*YCLNTH - HOVR2
      YVAL=0.6+I*0.2
      CALL NUMBER(-0.30,YLOC,H,YVAL,0., 1)
   20 CONTINUE
C
C.....LABEL THE X-AXIS
      CALL SYMBOL(3.0000,-0.6,HTIM2,'TOTAL MISSIONS',0.,14,1)
C
C.....LABEL THE Y AXIS
      CALL SYMBOL(-0.50,1.5,HTIM2,'CRACK LENGTH, INCHES',90.,20)
C
      RETURN
      END
```

```
      SUBROUTINE MTGRD2(XCLNTH,NXCYCL,YCLNTH,NYCYCL)
C
C.....GENERATE SEMI-LOG OPEN GRID, X( LOG )  Y, ( CARTESIAN ).
C
      DIMENSION GRID(9),GLNTH(9)
      DATA GRID/ 0.301030, 0.477121, 0.602060, 0.698970, 0.778151,
     1           0.845098, 0.903090, 0.954243, 1.0/
      DATA GLNTH/0.100, .160, .205, .235, .265, .285, .305,.325, .340/
      DATA H,HOVR2/0.0875, 0.044/
C
C.....FIND X-AXIS(HORIZONTAL) CYCLE LENGTH AND Y-AXIS(VERTICAL) HIGHT.
      YHIGHT=YCLNTH*NYCYCL
      XLGNTH=XCLNTH*NXCYCL
C
C.....DRAW THE OVERALL X-Y RECTANGLE, WITH ORIGIN AT (0., 0.).
      CALL PLOT(0.,0.,3)
      CALL PLOT(XLGNTH,0.,2)
      CALL PLOT(XLGNTH,YHIGHT,2)
      CALL PLOT(0.,YHIGHT,2)
      CALL PLOT(0.,0.,2)
C
C.....DRAW THE HORIZONTAL GRID LINES.
      NLINES=NYCYCL-1
      DO 10 I=1,NLINES
      YLOC=I*YCLNTH
      CALL PLOT(0.,YLOC,3)
C.....DRAW THE LEFT TIC MARK.
      CALL PLOT(HOVR2,YLOC,2)
C.....DRAW THE DOTTED LINE.
      CALL PLOT((XLGNTH-HOVR2),YLOC,2,1)
C.....DRAW THE RIGHT TIC MARK.
      CALL PLOT(XLGNTH,YLOC,2)
   10 CONTINUE
C
C.....DRAW THE LOGARITHMIC VERTICAL LINES.
      DO 20 NC=1,NXCYCL
      NCM1 = NC-1
      DO 30 I=1,9
      XLOC=(GRID(I)+ NCM1)*XCLNTH
      CALL PLOT(XLOC,0.,3)
      CALL PLOT(XLOC,GLNTH(I),2)
   30 CONTINUE
C.....DRAW THE DOTTED LINE AT EACH COMPLETE CYCLE.
      CALL PLOT(XLOC,(YHIGHT-HOVR2),2,1)
C.....DRAW THE TIC MARK.
      CALL PLOT(XLOC,YHIGHT,2)
   20 CONTINUE
C
      RETURN
      END
```

```
RELEASE 2.0              PRINT              DATE = 76314           16/32/20

      SUBROUTINE PRINT(SPNO,AI,PMAX,TEMP,R,FREQ, MISN, ALIFE,RR)
C
C.....GOULD PRINTS SALIENT PARAMETERS NEXT TO EACH PLOT.
      DIMENSION FREQ(2),RR(2)
      DATA H,HTIM2/0.0875, 0.1750/
      TWOA = 2.0 * AI
      X1=2.0
      X2=3.00
      X3=3.50
      X4=4.50
      X5=4.00
      Y=7.5
      CALL SYMBOL(X1,Y,H,'SPECIMEN NUMBER',0.,15,1)
      CALL SYMBOL(X2,Y,H,SPNO,0.,4,1)
C
C     CHECK IF MCT SPECIMEN OR SURFACE FLAW SPECIMEN.
      IF(AI .LT. 0.5) GO TO 58
      CALL SYMBOL(X3,Y,H,'MCT SPECIMEN',0.,12)
      GO TO 68
C
   58 CALL SYMBOL(X3,Y,H,'SURFACE FLAW',0.,12)
   68 Y=Y-HTIM2
      CALL SYMBOL(X1,Y,H,'INITIAL CRACK',0.,13,1)
      IF(AI .GT. 0.5) GO TO 56
      CALL NUMBER(X2,Y,H,TWOA,0.,3,1)
      GO TO 57
   56 CALL NUMBER(X2,Y,H,AI,0.,3,1)
   57 CALL SYMBOL(X3,Y,H,'INCHES',0.,6)
      Y=Y-HTIM2
      CALL SYMBOL(X1,Y,H,'MAX LOAD',0.,8,1)
      CALL NUMBER(X2,Y,H,PMAX,0.,3,1)
      CALL SYMBOL(X3,Y,H,'KIPS',0.,4)
      Y=Y-HTIM2
      CALL SYMBOL(X1,Y,H,'TEMPERATURE',0.,11,1)
      CALL NUMBER(X2,Y,H,TEMP,0.,-1,1)
      CALL SYMBOL(X3,Y,H,'DEGREES F',0.,9)
      Y=Y-HTIM2
      CALL SYMBOL(X1,Y,H,'STRESS RATIO',0.,12,1)
      CALL SYMBOL(X2,Y,H,RR,0.,8,1)
      Y=Y-HTIM2
      CALL SYMBOL(X1,Y,H,'FREQUENCY',0.,9,1)
      CALL SYMBOL(X2,Y,H,FREQ,0.,8,1)
      CALL SYMBOL(X3,Y,H,'MISSION',0.,7)
      Y=Y-HTIM2
      Y=Y-HTIM2
      CALL SYMBOL(X1,(Y+0.025),H,2,0.,-1,1)
      CALL SYMBOL(X2,Y,H,'ACTUAL DATA',0.,11,1)
      CALL NUMBER(X5,Y,H,ALIFE,0.,-1,1)
      CALL SYMBOL(X4,Y,H,'MISSIONS',0.,8)
      Y=Y-HTIM2
      CALL PLOT(1.75,Y,3)
      CALL PLOT(2.25,Y,2,-1)
      CALL SYMBOL(X2,Y,H,'PREDICTED',0.,9,1)
      CALL NUMBER(X5,Y,H,MISN,0.,-1,1)
      CALL SYMBOL(X4,Y,H,'MISSIONS',0.,8)
      RETURN
      END
```

```
      SUBROUTINE CFGRPH(CA,CN,A,XN,NPTS,NCALC,SPNO,AI,PMAX,TEMP,R,FREQ,
     1 MISN, ALIFE, RR, TYPE)
C.....PLOTS   CRACK-LENGTH, CYCLES ( A,N ) DATA AND PREDICTION.
      DIMENSION CA(1),CN(1),A(1),XN(1),TYPE(13)
      DIMENSION XNLOG(100),FREQ(2),RR(2)
      DATA XCLNTH,NXCYCL,  YCLNTH,NYCYCL/1.5, 4,   1., 6/
C
C.....COMPUTE THE STARTING INTEGER EXPONENT OF TEN.
C
      MINN=ALOG10( XN(1) )
      MINCN=ALOG10( CN(1) )
      MINX=MIN0( MINN, MINCN )
      XMIN=MINX
C
C.....DRAW THE LIFE PREDICTION SEMI-LOG GRID.
      CALL CFGRD(XCLNTH,NXCYCL,YCLNTH,NYCYCL,MINX,TYPE)
C
C.....PRINT THE SALIENT PARAMETERS.
      CALL PRINT(SPNO,AI,PMAX,TEMP,R,FREQ,MISN,ALIFE,RR)
C
C.....TAKE LOGS, COMPUTE THE SCALE PARAMETERS, PLOT DATA AND PREDICTION.
C
C.....TAKE LOGS OF X-PARAMETER.
      DO 10 I=1,NPTS
      XNLOG(I) = ALOG10( XN(I) )
   10 CONTINUE
C
C.....PLOT THE DATA.
      XNLOG(NPTS+1)=XMIN
      XNLOG(NPTS+2)=1./XCLNTH
      A(NPTS+1)=0.0
      A(NPTS+2)=0.1/YCLNTH
      CALL LINE(XNLOG,A,NPTS,1,-1,2)
C
C.....TAKE LOGS OF X-PARAMETER.
      DO 20 I=1,NCALC
      XNLOG(I) = ALOG10( CN(I) )
   20 CONTINUE
C
C.....DRAW THE PREDICTION.
      XNLOG(NCALC+1)=XMIN
      XNLOG(NCALC+2)=1./XCLNTH
      CA(NCALC+1)=0.0
      CA(NCALC+2)=0.1/YCLNTH
      CALL LINE(XNLOG,CA,NCALC,1,0,0)
C
      RETURN
      END
```

```
      SUBROUTINE CFGRD(XCLNTH,NXCYCL,YCLNTH,NYCYCL,MINX,TYPE)
      DIMENSION TYPE(13)
C
C     PLOT AND ANNOTATE A SEMI-LOG LIFE PREDICTION GRID.
C
      DATA H,HOVR2/0.0875, 0.044/
      HTIM2=H*2
C
C      DRAW THE GRID
      CALL MTGRD2(XCLNTH,NXCYCL,YCLNTH,NYCYCL)
C
C.....DRAW THE Z-SHAPED INTERRUPTION.
C     CALL PLOT(0.,0.2,3)
C     CALL ERASE(1)
C     CALL PLOT(0.,0.5,2)
C     CALL ERASE(0)
C     CALL PLOT(-0.3,0.35,2)
C     CALL PLOT(+0.3,0.35,2)
C     CALL PLOT(0.,0.2,2)
C
C.....WRITE FIGURE AND TITLE
      TOP=8.7
      CALL SYMBOL(3.0,(TOP+3.*H),HTIM2,'P&WA MATERIALS AND MECHANICS LAB
     1ORATORY',0.,39,1)
      CALL SYMBOL(3.0,TOP,HTIM2,'AFML CONTRACT  F33615-75-C-5097',
     1             0.,31,1)
      CALL SYMBOL(3.0,(TOP-3.*H),H,'FIGURE    ',0.,9,1)
      CALL SYMBOL(3.0,(TOP-5.*H),H,'SURFACE FLAW SPECIMEN CRACK GROWTH P
     1REDICTION',0.,45,1)
      CALL SYMBOL(3.0,(TOP-7.*H),H,'INTERPOLATIVE HYPERBOLIC SINE MODEL'
     1,0.,35,1)
      CALL SYMBOL(3.0,(TOP-9.*H),H,TYPE,0.,52,1)
C
C.... ANNOTATE THE X-AXIS
      YLOC=-0.25
      DO 10 I=1,3
      XLOC=I*XCLNTH
      CALL NUMBER(XLOC,YLOC,H,10,0.,-1,1)
      CALL NUMBER((XLOC+HTIM2),(YLOC+H),H,(MINX+I),0.,-1,1)
   10 CONTINUE
C
C.....ANNOTATE THE Y-AXIS
      CALL NUMBER(-0.30,0.,H,0.,0., 1)
      DO 20 I=1,5
      YLOC=I*YCLNTH - HOVR2
      YVAL=I*0.1
      CALL NUMBER(-0.30,YLOC,H,YVAL,0., 1)
   20 CONTINUE
C
C.....LABEL THE X-AXIS
      CALL SYMBOL(3.0000,-0.6,HTIM2,'TOTAL MISSIONS',0.,14,1)
C
C.....LABEL THE Y AXIS
      CALL SYMBOL(-0.50,1.0,HTIM2,'HALF CRACK LENGTH, INCHES',90.,25)
C
      RETURN
      END
```

APPENDIX C
SAMPLE INPUT DATA

```
SPECIMEN 653  MCT SPECIMEN FOR AFML FRACTURE MECHANICS.
  LUKE        CENTER FREQUENCY IN THE FIRST 8 SPACES OF THIS LINE.
  MIXED       CENTER STRESS RATIO IN THE FIRST 8 SPACES OF THIS LINE.
                 RETARDATION MODEL                       CENTER IN 52 SPACES.
 &INPUT  PERIOD =   1.,  .0333,  1.,  .0350, 0.0136, 1.,   .0350, .0285,
                    1.,  .0333,
         NPER   =   1,     1,    1,     1,     1,     7,     1,     3,
                    1,     1,
         COEF1 = 0.500, 0.500, 0.500, 0.500, 0.500, 0.500, 0.500, 0.500,
                 0.500, 0.500,
         COEF2 = 3.938, 4.297, 4.116, 4.378, 4.297, 4.169, 4.292, 4.297,
                 4.116, 4.297,
         COEF3 =-1.397,-1.479,-1.305,-1.633,-1.479,-1.277,-1.565,-1.479,
                -1.305,-1.479,
         COEF4 =-4.156,-2.519,-4.241,-2.998,-2.519,-4.267,-2.807,-2.519,
                -4.241,-2.519,
         PMAX =  2.760, 2.760, 6.000, 4.680, 4.680, 5.340, 4.680, 4.680,
                 6.000, 2.760,
         RATIO = 0.2,   1.0,   0.46,  1.0,   1.0,   0.52,  1.0,   1.0,
                 0.46,  1.0,
         AI = 1.00830,  AF = 1.7500,  TEMP = 1200.,  PKLD = 6.0,
         WID = 2.5090,  THIK = 0.8510,
         LOOPS = 10,
 &END
 1.0063        1.
 1.0097        5.
 1.0235       17.
 1.0344       29.
 1.0429       41.
 1.0465       51.
 1.0562       61.
 1.0606       71.
 1.0639       81.
 1.0704       91.
 1.0773      101.
 1.0811      111.
 1.0859      120.
 1.0933      129.
 1.0997      138.
 1.1026      147.
 1.1153      170.
 1.1266      193.
 1.1465      230.
 1.1640      260.
 1.1797      290.
 1.1965      319.
 1.2125      348.
 1.2328      384.
 1.2692      440.
 1.2873      470.
 1.3074      500.
 1.3281      530.
 1.3544      560.
 1.3769      590.
 1.3992      617.
 1.4292      650.
 1.4795      682.
 1.4810      699.
 1.5152      721.
 1.5394      742.
```

1.5734	758.
1.5997	770.
1.6173	781.
1.6401	790.
1.6575	799.
1.6896	808.
1.7175	815.
-1.0	

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